Forecasting the Discounts of Market Prices from Appraised Values for Real Estate Limited Partnerships

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A vexing problem for the appraisal industry has been estimating an appropriate discount for the value of real estate limited partnerships (RELPs) relative to their appraised value. This research develops a linear regression model that explains over 80% of the cross-sectional variation in discounts across 60 RELPs using characteristics of each partnership. Among a holdout sample of 41 RELPs, the model provides forecasts of discounts that are superior to assuming no discount or applying a mean discount to all partnerships. Discounts are greatest for RELPs with low current yields, low leverage and high trading ranges for their market prices.

The appraisal industry is generally quite good at assessing the market value of real estate properties. Unfortunately, this skill alone is insufficient when it comes to the valuation of real estate limited partnership units. Real estate limited partnership units (RELPs) are traded among a loose affiliation of brokerage firms and specialized dealers and have historically traded at substantial discounts to their appraised value per unit. For example, the May/June 1993 issue of The Perspective, a bimonthly publication of Partnership Profiles Inc., documents a mean discount of 46% in a sample of 117 RELPs. Similarly, this research documents a mean discount of 45% for a sample of 112 RELPs (see Figure 1). Thus, appraising the properties held by the partnership is a necessary, but not sufficient, step in determining the value of the limited partnership units.

Providing a realistic estimate of the discounts of limited partnership units to their appraised value is particularly important in light of the tax court ruling in Estate of Berg v. Comm'r (61 T.C.M. 2949, 1991). In this case, the tax court allowed a "minority interest" discount of 30% for a closely held real estate holding company based on discount data from publicly traded real estate entities. The analysis presented to the court in the Estate of Berg (61 T.C.M. 2957, 1991) was "particularly persuasive because [it] relies on very

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specific studies of comparable properties, and then adjusts the minority interest discount for the relevant factors of decedent’s interest.”

The purpose of this research is to use standard statistical techniques to develop a simple linear model of the level of discounts for RELPs relative to their appraised value. The results of the analysis are quite promising. Based on readily observable characteristics of real estate partnerships, over 80% of the cross-sectional variation in limited partnership discounts is explained. In addition, the statistical model developed is far superior in its ability to forecast RELP discounts than, for example, applying a mean discount to all partnerships.

The remainder of this paper is organized as follows. It begins with a discussion of the factors that affect the discounts of RELPs. A discussion of the data and methods employed is followed by a summary of the empirical results.

Factors Affecting Limited Partnership Discounts

The factors that might affect the level of RELP discounts are summarized. There are three possible explanations for why RELPs trade at a discount to their appraised values: (1) agency costs; (2) taxes; and (3) liquidity.
Here the appraised value is assumed to represent the "true" value of the underlying property. Under this assumption, the observed market prices of RELPs represent a discount from fundamental (market) value. Alternatively, it is possible that appraisals are inflated and, thus, an overestimate of the fundamental (market) value of the properties. Whether appraisals represent a reliable estimate of fundamental value or are inflated affects the interpretation of the results documented in this research. However, ultimately the goal was to develop a simple and robust method of estimating market prices based on characteristics of individual partnerships. This goal was motivated by the insistence of the tax courts for a systematic analysis of the discount of market prices from their appraised value. Whether these market prices represent a discount from fundamental value or a "mark-down" from inflated appraisals is a moot issue for this purpose.

Agency Costs

RELPs are organized such that the general partners control the day-to-day operations of the partnership, while the limited partners have little involvement with these operations. Furthermore, the RELPs trade through a loose affiliation of brokerage firms and specialized dealers. As such, in contrast to public corporations or real estate investment trusts (REITs) which are traded on major stock exchanges, RELPs are not subject to the same close scrutiny of analysts and other members of the investment community. This environment exacerbates the asymmetry of information between the general partners and the limited partners. In contrast to the RELPs studied here, in a sample of 77 exchange-traded REITs between 1982 and 1989, Damodaran and Liu (1993) document that the market prices of REITs trade, on average, near or slightly above their appraised value. However, using a sample of 75 equity REITs over the period 1985–1992, Capozza and Lee (1995) document that REITs traded at an average discount of 8% to their net asset value, though the discounts were larger in the last half of the sample period.

Jensen (1986) argues that managers of firms with high levels of "free cash flow" and poor investment opportunities have an incentive to squander these excess cash flows on poor investments or perquisites. There is accumulating evidence in the empirical literature in finance that these agency costs have an impact on firm value. Lehn and Poulsen (1989) document that one source of gains in going private transactions is the mitigation of agency costs associated with excessive free cash flow. Lang, Stulz and Walkling (1989) document that the total gains from acquisition are largest when a well-managed bidder acquires a poorly managed target. Smith and Kim (1994) further document that the gains from acquisition are highest when firms with little financial slack combine with firms with excessive free cash flow.
These arguments are particularly relevant for the general partners of RELPs. With large asymmetries of information between the general partners and limited partners, the potential for opportunistic behavior by general partners is high. The agency costs associated with this opportunistic behavior can explain why RELPs trade at a discount to their appraised value. Three mechanisms general partners can use to reduce the agency costs associated with the asymmetry of information between general partners and limited partners includes: (1) cash distributions or current yield; (2) leverage; and (3) operating performance.

Current Yield. Jensen (1986) argues that managers of firms with free cash flow can reduce the attendant agency costs by developing an implicit contract with stockholders by pursuing a high dividend distribution policy. Lang and Litzenberger (1989) argue that if managers are overinvesting, an increase in the dividend will reduce the overinvestment and increase the market value of the firm. Consistent with this hypothesis, they document that firms with poor investment opportunities (as measured by Tobin's $q$) have the largest gains in market value when there is a significant increase in the firm's dividend. Wang, Erickson and Gau (1993) provide strong evidence to support the hypothesis that REIT dividend policies are at least partially determined by agency costs and that high payout ratios are preferred by shareholders because they use capital markets as a monitoring device for management investment decisions.

In the case of RELPs, a high current yield can serve to reduce the agency costs described above. General partners stand to lose management fees if their partnerships are liquidated. Consequently, general partners have an incentive not to liquidate their partnerships. In fact, general partners have an incentive to build their empires by retaining the earnings of properties for further investment—thus increasing the management fees due to them. Of course, investors in limited partnerships are (or should be) aware of these incentive problems. Consequently, general partners must establish their reputation in a credible fashion. One means of establishing the integrity of a general partner is for the general partner to pay out earnings to limited partners, rather than investing in additional properties. Thus, investors in limited partnerships do not desire current yield because it is preferred to capital gains, but rather because the current yield provides a credible indication of the integrity of the general partner.

It is commonly perceived that the level of discount is affected by the distribution policy of the partnership. The May/June 1993 issue of *The Perspective*, a common source for limited partnership information, summarizes this position succinctly: “The name of the game in the LP
secondary market is spelled Y-I-E-L-D. Very few buyers are willing to wait years and years before realizing any cash return on their investment, especially with so many general partners unwilling to liquidate their partnership anytime soon.”

Several authors have conjectured about a relation between discounts and current yields (Hall 1989, 1993; and Korn and Hitchner 1993). However, there has been no systematic study of the relation between discounts and current yields.

Leverage. Jensen (1986) also argues that the use of debt can serve as a means of reducing the agency costs associated with excessive free cash flow, since the interest expense associated with debt reduces the available free cash flow. Furthermore, debtholders are concerned with a creditor’s ability to make timely interest and principal payments and thus serve an additional monitoring role. In the case of RELPs, partnerships with high levels of debt have been judged creditworthy by lenders. In addition, the high levels of debt suggest that the opportunity for managerial malfeasance by the general partners is reduced since lenders are generally careful in their monitoring of creditors. Thus, as is the case with current yields, high levels of leverage may provide a signal about the managerial ability of the general partner. This line of reasoning suggests that partnerships with high levels of leverage would have lower discounts.

Certainly, not all agree that discounts should decline with the increased use of debt. The May/June 1993 issue of The Perspective suggests that the discounts are higher for partnerships “saddled with high debt loads.” Though debt may increase the scrutiny under which general partners operate, it may also impose potential costs associated with financial distress. Ultimately, whether debt affects the level of the RELP discount is an empirical question that this research addresses.

Operating Performance. Hall (1989; p.177) argues that “higher revenues and earnings possibly provide a signal to the investor indicating the likelihood of a professionally managed, successful real estate investment.” Korn and Hitchner (1993) also cite the “quality of the property and the likelihood of continued earning power” as a relevant factor to consider in the quantification of the discount. If operating performance is related to managerial ability, investors would pay a premium for limited partnerships with superior operating performance. These arguments indicate that the operating performance of a RELP provides a signal about managerial ability and yields the testable hypothesis that discounts are lower for RELPs with superior operating performance.
Taxes

Partnerships often accrue, but do not pay, taxes on capital gains. These capital gains taxes are deferred until a property is sold. This deferral of capital gains taxation may affect the value of limited partnership units. Consider a property that was purchased at cost $C$. The market value of this property is $M$. Capital gains are taxed at a rate $\tau$. Thus, the value of this property ($P$), after accounting for taxes, is the market value less any tax liability:

$$P = M - \tau(M - C).$$

The after-tax value of the property decreases with the size of the accumulated capital gain tax liability, $\tau(M - C)$.

A simple example illustrates the impact of taxation on partnership units. A property purchased for $1$ million is currently worth $2$ million. Capital gains are taxed at a rate of $30\%$. There are $2000$ partnership units being sold on this property.¹ These partnership units are worth $850 each: $(2,000,000 - .30(2,000,000 - 1,000,000))/2000$. In contrast, if the property were purchased for $2,000,000$ the partnership units would sell for $1,000 each, solely because of the reduced capital gains tax: $(2,000,000 - .30(2,000,000 - 2,000,000))/2000$.

Note the impact that taxation has on the observed discount. In the above example, in both situations the appraised value of the property is $1,000$ per unit. When there are embedded capital gains taxes, the former case, the limited partnership units sell at a discount. When there are no capital gains taxes, the latter case, the limited partnership units sell at their appraised value. In fact, if the property were selling below its acquisition price ($M < C$), the limited partnership units could sell at a premium because of the potential tax deductibility of the capital losses.

In sum, the discounts of limited partnership units to their appraised value should increase with the size of the embedded capital gains tax liability. Some have referred to this tax-related discount as "trapped capital gains" (Korn and Hitchner 1993) or "unrealized capital gains" (Hall 1989). Similar arguments have been made to explain the discount of the market price of closed-end mutual funds to their net asset value. Malkiel (1977) provides weak evidence that a portion of the discounts of closed-end mutual funds

¹ The role of the general partner is ignored for the sake of simplicity.
can be explained by unrealized taxes on capital gains. More recently, however, Brickley, Manaster and Schallheim (1991) document a negative relationship between closed-end fund discounts and unrealized capital gains and a positive correlation between the discounts and unrealized capital losses. These results are opposite conventional wisdom that unrealized gains represent a tax liability that increases the discount and are inconsistent with the empirical evidence of Malkiel (1977). Whether unrealized capital gains are an important determinant in the valuation of RELPs remains an empirical issue that the current study addresses.

Liquidity

Amihud and Mendelson (1986) develop a model in which they document that expected returns are negatively related to the liquidity of an asset. One implication of their model is that an asset’s value can increase by increasing the liquidity of the asset. RELPs, which are traded in an unorganized secondary market, are relatively illiquid assets. The illiquidity hypothesis predicts that RELPs with less liquidity would trade at a larger discount to their appraised value. In justifying the discounts of limited partnership units, Korn and Hitchner (1993; p. 238) argue that “an asset that is very difficult to sell is certainly worth significantly less than an identical one that is readily marketable.” Hall (1989) refers to this as the “lack of marketability” discount. If the market for limited partnership units is small and investors wishing to liquidate their holdings must incur significant costs in the search for a willing buyer, limited partnership units would sell at a discount to their appraised value. The size of this discount would depend on the liquidity of the individual partnership units.

Data and Methods

Data

To investigate the impact of each of the factors discussed, a final sample of 112 RELPs was examined. Several steps were involved in the identification of this sample. Initially, 155 real estate partnerships with net asset values (appraised values) per unit reported in the May/June 1993 issue of The Perspective were evaluated. These appraised values represent the values reported by the partnership as of December 31, 1992. Partnership Profiles, Inc., the publisher of The Perspective, states that the value per unit represents the estimated intrinsic value per unit outstanding of the partnership as reported by the general partner based on the estimated value of the partnership’s assets less liabilities. These estimates of value per unit are made by
either a general partner or a third party unaffiliated with the partnership. In the final sample of 112 real estate partnerships, 36% of the appraisals were performed by the general partner.

The high and low trading price for partnership units between December 1, 1992 and January 31, 1993 ($P^h_i$ and $P^l_i$, respectively, with the subscript $i$ denoting the $i^{th}$ partnership) were identified in the January/February 1993 issue of The Perspective. Trade price information was gathered from fourteen firms (for example, Chicago Partnership Board and Churchill Securities) that trade the limited partnership units. From the original sample of 155 partnerships, nonzero appraisal values and trading prices were identified for 128 partnerships. From this sample, fourteen mortgage limited partnerships were deleted, since mortgage limited partnerships invest in mortgages rather than real estate, and two outlying observations\(^2\) leaving 112 partnerships.

To calculate the level of the discount for each partnership ($D_i$), the average of the high and low trading prices was divided by the appraised value per unit ($NAV_i$) to yield a percentage discount:

$$D_i = -\left(\frac{(P^h_i + P^l_i)/2}{NAV_i} - 1\right).$$  \hspace{1cm} (2)$$

The minus sign yields discounts stated in positive values. Thus, for example, a limited partnership with an appraised value per unit of $1000 and a high/low trading price of $600 trades at a discount of 40%. In the empirical analyses that follow, the following measure is considered, which is refered to as log discount:

$$-\ln(1 - D_i) = -\ln \left(\frac{(P^h_i + P^l_i)/2}{NAV_i}\right).$$  \hspace{1cm} (3)$$

Finally, measures were developed for each of the variables discussed in the

\(^2\) Preliminary analyses revealed two partnerships that for no apparent reason traded at substantial premiums to their net asset value. These partnerships are both managed by Damson Birtcher (Realty Income I and Real Estate Partners III). Spencer Jefferies, the editor of The Perspective, indicated that there was a discrepancy in the number of units used to calculate the price per unit and the appraised value per unit. These partnerships are excluded from the analyses, though their inclusion does not materially alter any of the conclusions. Of the remaining 112 partnerships, 3 trade at a premium to their appraisal value per unit, with the largest premium being 35% (Corporate Property Associates 2).
prior section. The current yield of each partnership unit \((CY)\), obtained from the January/February 1993 issue of *The Perspective*, is the annualized regular cash distribution rate of the partnership per unit as of December 31, 1992 scaled by the appraised value per unit. The leverage ratio was defined as the secured debt outstanding (net of discounts for reporting purposes) as a percentage of the partnership’s property at cost. The operating performance of each partnership unit was measured by two variables. First, the ability of the property to generate revenue \((R)\) was measured as the gross revenues of the partnership divided by the properties at cost. Gross revenues were obtained from the partnership’s financial statements and measured the revenue generated by the properties, but did not include gains or losses from property dispositions, refinancings or other extraordinary events. Second, the ability of the property to generate earnings \((E)\) was measured as the operating surplus (or deficit) divided by the properties at cost. Operating surplus was estimated based on the net income (or loss) reported by the partnership for the period reviewed before gains or losses on property dispositions, refinancings and other extraordinary events. \(^3\) The secured debt, gross revenue, operating surplus and property at cost information was obtained from *Partnership Profiles*, Fall 1993, Volumes I and II and represents the most recently reported fiscal results for each partnership.

The unrealized capital gains for each partnership was estimated as the appraised value per unit less the book value per unit, scaled by the appraised value per unit. The book value per unit appeared in *Partnership Profiles*. \(^4\) This was an estimate of the unrealized capital gains, since the book value per unit is that used for reporting purposes and could differ from that used for tax purposes.

The liquidity or marketability of each partnership was difficult to measure directly. Two measures of liquidity were employed. First, the number of partnership units traded for each partnership \((NT)\) between December 1, 1992 and January 31, 1993 was used as one proxy for liquidity. Second, the trading range \((TR)\), measured as the difference between the high and low trading price scaled by the average of the high and low trading price was

\(^3\) Several adjustments to net income were made in order to arrive at operating surplus. The details of these adjustments are provided by Partnership Profiles, but generally entail adding back noncash items such as depreciation and deferred expense, while deducting noncash items such as deferred revenue.

\(^4\) Book value per unit is derived on the cost basis of the partnership’s tangible assets (before depreciation and amortization but net of loss allowances) less its actual liabilities (after discounts including minority interests) as stated on the partnership’s balance sheet.
used as another liquidity proxy. The trading range provides an estimate of the cost of liquidity. These data were reported in the January/February 1993 issue of The Perspective.

Descriptive statistics on each of the variables are presented in Table 1; a correlation matrix of the variables is presented in Table 2. The percentage discount of the real estate limited partnerships studied here averages 45.4%, with a median discount of 45.7%. Thus, as is widely observed, this sample reveals that limited partnerships trade at a substantial discount to their appraised value per unit. The distribution of the percentage discount is graphed in Figure 1. Standard tests that the distribution of the percentage discount is normal fail to reject the null hypothesis of normality. It is interesting to note that the level of discounts for RELPs is much larger than the level of discounts documented by Damoradan and Liu (1993) and Capozza and Lee

<p>| Table 1 | Descriptive statistics on selected characteristics of RELPs: December, 1992. |</p>
<table>
<thead>
<tr>
<th>Variable Description</th>
<th>Symbol</th>
<th>Mean</th>
<th>Median</th>
<th>Min.</th>
<th>Max.</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount (%)</td>
<td>(D_i)</td>
<td>45.4</td>
<td>45.7</td>
<td>-35.1</td>
<td>94.1</td>
<td>23.7</td>
</tr>
<tr>
<td>Log Discount (%)</td>
<td>(-ln(1 - D_i))</td>
<td>71.7</td>
<td>61.1</td>
<td>-30.1</td>
<td>282.5</td>
<td>51.7</td>
</tr>
<tr>
<td>Log Price</td>
<td>(ln(P_i))</td>
<td>4.9</td>
<td>5.0</td>
<td>.6</td>
<td>6.8</td>
<td>1.3</td>
</tr>
<tr>
<td>Log NAV</td>
<td>(ln(NAV_i))</td>
<td>5.6</td>
<td>5.9</td>
<td>2.1</td>
<td>7.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Current Yield (%)</td>
<td>(CY_i)</td>
<td>5.2</td>
<td>5.7</td>
<td>.0</td>
<td>16.2</td>
<td>3.9</td>
</tr>
<tr>
<td>Leverage (%)</td>
<td>(L_i)</td>
<td>14.4</td>
<td>.0</td>
<td>0</td>
<td>70.0</td>
<td>21.2</td>
</tr>
<tr>
<td>Revenue Performance (%)</td>
<td>(R_i)</td>
<td>15.5</td>
<td>12.9</td>
<td>7.5</td>
<td>109.8</td>
<td>11.4</td>
</tr>
<tr>
<td>Earnings Performance (%)</td>
<td>(E_i)</td>
<td>.2</td>
<td>2.2</td>
<td>-38.5</td>
<td>40.5</td>
<td>11.2</td>
</tr>
<tr>
<td>Unrealized Cap. Gains (%)</td>
<td>(UCG_i)</td>
<td>30.0</td>
<td>9.0</td>
<td>-30.3</td>
<td>384.0</td>
<td>56.9</td>
</tr>
<tr>
<td>Number of Trades</td>
<td>(NT_i)</td>
<td>4.3</td>
<td>3.0</td>
<td>1.0</td>
<td>19.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Trading Range (%)</td>
<td>(TR_i)</td>
<td>13.8</td>
<td>11.3</td>
<td>.0</td>
<td>64.9</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Discount to appraised value is calculated using the average of high and low trading prices less appraised value per unit divided by appraised value per unit. Discounts are expressed as positive values. Log Discount is minus the natural log of the ratio of appraised value per unit to the average of the high and low trading price. Log Price is the natural log of high and low trading price. Log NAV is the natural log of the appraised value per unit. Current yield is the annualized regular cash distribution rate of the partnership per unit divided by appraised value per unit. Leverage is the ratio of secured debt to property at cost. Revenue (earnings) performance is measured as gross revenues (operating surplus) divided by property at cost. Unrealized capital gains are estimates of capital gains taxes that would be incurred if the property was liquidated scaled by the appraised value per unit. Number of trades represent the number of partnership units traded from 12/1/92 through 1/31/93. Trading range is the ratio of the difference between high and low trading price to the average of the high and low trading price. The number of observations varies from 98 to 112 depending on data availability.
Table 2: Pearson Correlation (%) for selected characteristics of RELPs: December, 1992.

<table>
<thead>
<tr>
<th>Variable Description</th>
<th>Symbol</th>
<th>$D_i$</th>
<th>$ln(1 + D_i)$</th>
<th>$ln(P_i)$</th>
<th>$ln(NAV_i)$</th>
<th>$CY_i$</th>
<th>$L_i$</th>
<th>$R_i$</th>
<th>$E_i$</th>
<th>$UCG_i$</th>
<th>$NT_i$</th>
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</thead>
<tbody>
<tr>
<td>Discount</td>
<td>$D_i$</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Discount</td>
<td>$-ln(1 - D_i)$</td>
<td>94.2*</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Price</td>
<td>$ln(P_i)$</td>
<td>-53.0*</td>
<td>-50.8*</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log NAV</td>
<td>$ln(NAV_i)$</td>
<td>-17.8</td>
<td>-12.5</td>
<td>91.8*</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Yield</td>
<td>$CY_i$</td>
<td>-75.7*</td>
<td>-71.0*</td>
<td>57.0*</td>
<td>33.0*</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>$L_i$</td>
<td>14.2</td>
<td>21.6*</td>
<td>-10.3</td>
<td>-2.3</td>
<td>-34.1*</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Revenue Performance</td>
<td>$R_i$</td>
<td>4.6</td>
<td>6.5</td>
<td>-19.5</td>
<td>-20.2*</td>
<td>-5.4</td>
<td>-7.1</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Earnings Performance</td>
<td>$E_i$</td>
<td>-36.7*</td>
<td>-31.4*</td>
<td>13.2</td>
<td>0.8</td>
<td>34.4*</td>
<td>-10.4</td>
<td>51.2*</td>
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<td></td>
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<tr>
<td>Unrealized Capital Gains</td>
<td>$UCG_i$</td>
<td>30.6*</td>
<td>31.3*</td>
<td>-37.6*</td>
<td>-30.0*</td>
<td>-50.3*</td>
<td>41.9*</td>
<td>-15.4</td>
<td>-37.4*</td>
<td>100.0</td>
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<tr>
<td>Number of Trades</td>
<td>$NT_i$</td>
<td>-10.1</td>
<td>-15.8</td>
<td>15.2</td>
<td>10.3</td>
<td>15.9</td>
<td>-35.9*</td>
<td>8.9</td>
<td>-16.9</td>
<td>-24.1*</td>
<td>100.0</td>
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<tr>
<td>Trading Range</td>
<td>$TR_i$</td>
<td>1.5</td>
<td>-3.4</td>
<td>10.9</td>
<td>11.0</td>
<td>7.8</td>
<td>-31.2*</td>
<td>-2.0</td>
<td>14.1</td>
<td>-24.2*</td>
<td>67.3*</td>
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</table>

Discount to appraised value is calculated using the average of high and low trading prices less appraised value per unit divided by appraised value per unit. Discounts are expressed as positive values. Log Discount is the natural log of the ratio of appraised value per unit to the average of the high and low trading price. Log Price is minus the natural log of high and low trading price. Log NAV is the natural log of the appraised value per unit. Current yield is the annualized regular cash distribution rate of the partnership per unit divided by appraised value per unit. Leverage is the ratio of secured debt to property at cost. Revenue (earnings) performance is measured as gross revenues (operating surplus) divided by property at cost. Unrealized capital gains are estimates of capital gains taxes that would be incurred if the property was liquidated scaled by the appraised value per unit. Number of trades represent the number of partnership units traded from 12/1/92 through 1/31/93. Trading range is the ratio of the difference between high and low trading price to the average of the high and low trading price. Each of the variables summarized in this table are discussed in detail in the data and methods section of this paper. Correlations are based on the full sample of 112 RELPs. The number of observations for the pairwise correlations range from 98 to 112 depending on data availability.

* Significant at the 5% level, two-sided test-statistic.
(1995) for REITs. The higher level of discounts for RELPs is consistent with higher agency costs and lower liquidity for RELPs relative to REITs, which are traded on major stock exchanges and followed more closely by analysts and the investment community.

In addition to the level of the percentage discount, four additional observations emerge from the descriptive statistics. First, the earnings performance of these partnerships was less than spectacular. The average partnership earned close to nothing or .2% of the cost of the property. The median performance was 2.2%. Second, on average, these limited partnerships have some unrealized capital gains. More than 50% have capital gains in excess of 9.0% of the appraised value per unit. These unrealized capital gains alone, however, are not large enough to justify an average discount in excess of 40%. Third, the partnership units trade infrequently. More than half of the units had less than four trades from 12/1/92 through 1/31/93. Fourth, over half of these partnerships employed no secured debt to finance their operations.

Methodology

Estimation Sample. To evaluate the impact of each of the variables on the level of discount for a particular partnership, two steps were used. First, the linear cross-sectional regression model of RELP discounts was estimated:

\[ D_i = a + b_1 CY_i + b_2 L_i + b_3 R_i + b_4 E_i + b_5 UCG_i + b_6 NT_i + b_7 TR_i + b_8 APP_i + e_i, \]  

(4)

where \( a \) and \( b_1 \) through \( b_8 \) are model parameters and \( e_i \) is an error term. Each of the independent variables correspond to those discussed thus far, with the exception of \( APP_i \). This variable represents a dummy variable that takes on a value of one if the appraisal was performed by an independent third party and a value of zero if the appraisal was performed by a general partner. Including this variable in the regression model allowed evaluation of the level of the discount as it related to the party performing the appraisal. Formal statistical tests that assessed the impact of each variable on the level of the observed discount involved assessing whether the values of the slope coefficients in the regression model, \( b_1 \) through \( b_8 \), were significantly different from zero. This regression model was estimated using a subsample of 60 of the 112 partnerships. The 60 RELPs were selected by arranging all the RELPs in alphabetical order and selecting the first 60 RELPs with available data. The remaining partnerships were reserved as a "holdout" sample.
Due to data constraints (primarily on the availability of leverage data), the holdout sample ultimately contained 41 observations. To test the robustness of the results using the percentage discount as the dependent variable, a second regression was estimated using the log discount \(-\ln(1 - D_i)\) as the dependent variable.

One complication was in the interpretation of the coefficient estimates. The estimated regressions provided an accurate assessment of the deviations from appraised values. However, if appraised values are interpreted as a measure of the fundamental value of the RELP, the problem of errors-in-the-variables arises. Since appraised values measure fundamental values with errors, estimated regression coefficients can be biased. Furthermore, since the appraised values are used to measure the dependent variable and some independent variables in the regressions, there is an attenuation bias (slope coefficients are biased towards zero) and a positive bias (slope coefficients are biased away from zero), making the interpretation of the regression coefficients problematic.

To mitigate the errors-in-the-variables problem, a third regression was estimated where the dependent variable was the log of the market price (measured as the average of the high and low trading price). By specifying the dependent variable as the log market price, in lieu of the discount, the variables measured using the appraisal values could be moved to the right-hand side of the regression equation. Thus, the positive bias associated with the errors-in-the-variables is alleviated, though the attenuation bias remains.

A second approach for alleviating the errors-in-the-variables problem was to use the ranks of the variables measured with error as instrumental variables. For a thorough discussion of this procedure, consult Greene (1993; p. 284). In the interest of parsimony, the full results of this analysis are not reported. However, when the ranks are used as instrumental variables, none of the substantive conclusions of the analysis are altered.

In sum, three comments are relevant when considering the problem of errors-in-the-variables. First, measurement error is only an issue when appraised values are considered an estimate of fundamental value. When the results reported in this research are interpreted as deviations from appraised value, the measurement error problem is not relevant since appraised values measure appraised values without error. This is particularly important since a primary motivation of this paper was arriving at an estimate of RELP market value based on their appraised values for tax reporting purposes. Second, by isolating the variables which are measured with error on the right-hand side of the regression model (as is done in the third model
specification), the positive bias resulting from the errors-in-the-variables problem was alleviated. Third, when the ranks were used as instruments for the variables measured with error, none of the substantive conclusions of this research were affected.

**Holdout Sample.** Since a primary motivation of this research was to provide an accurate forecast of the discount of market prices from appraised values, the ability of the estimated regression model was tested on a holdout sample of 41 RELPs. In order to assess the ability of the regression model to forecast discounts, the coefficient estimates from the percentage discount model were used to forecast the discount for partnerships forming the holdout sample (those partnerships not among the 60 used to estimate the above regression model). This analysis provides an estimate of the reliability of forecasts of percentage discounts from appraised values.

**Results**

**Model Estimation**

The results of the estimation of the three regression models based on the sample of 60 real estate limited partnerships are presented in Table 3. Of the variables considered, four emerge as important factors influencing the discounts of RELPs: (1) current yield; (2) leverage; (3) trading range; and (4) whether the appraisal was performed by a general partner or independent third party. However, two of these variables, leverage and the identity of the appraiser, are sensitive to the model specification. An analysis of the residuals of the three models indicate that the standard assumptions involved in the estimation of the regression were maintained (for example, homoscedastic and normally distributed error terms). The two models based on the percentage discount and log discount both explain over 70% of the cross-sectional variation in the discounts of RELPs (see adjusted $R^2$, Table 3). This relatively high explanatory power bodes well for the possibility of forecasting the level of the discount with a reasonable degree of accuracy. The third model, which employs the negative log of market price as the dependent variable to mitigate the problems associated with measurement error, confirms that current yield and trading range are important determinants of the cross-sectional variation in RELP prices.

Of the variables considered, the current yield of the limited partnership is clearly the most important determinant of the level of discount relative to the partnership's appraised value. The results indicate that a 100 basis point increase in the current yield corresponds with a 586 basis point _decrease_ in the partnership percentage discount. Thus, for example, moving from the
Table 3 - Estimated regression models of percentage discount, log discount and log price regressed on selected characteristics of RELPs: December, 1992.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Discount (D_i)</th>
<th>Log Discount (-\ln(1 + D_i))</th>
<th>Log Price (-\ln\left(\frac{P_i^L}{P_i^H}\right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Symbol</td>
<td>Coef</td>
<td>Coef</td>
</tr>
<tr>
<td>Intercept</td>
<td>-</td>
<td>.70</td>
<td>11.5**</td>
</tr>
<tr>
<td>Log NAV</td>
<td>(\ln(NAV_i))</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Current Yield</td>
<td>(CY_i)</td>
<td>-5.86</td>
<td>-14.6**</td>
</tr>
<tr>
<td>Leverage</td>
<td>(L_i)</td>
<td>-.20</td>
<td>-3.0**</td>
</tr>
<tr>
<td>Revenue Performance</td>
<td>(R_i)</td>
<td>.31</td>
<td>1.1</td>
</tr>
<tr>
<td>Earnings Performance</td>
<td>(E_i)</td>
<td>-.31</td>
<td>-1.7</td>
</tr>
<tr>
<td>Unrealized Capital Gains</td>
<td>(U_CG_i)</td>
<td>-.03</td>
<td>-1.1</td>
</tr>
<tr>
<td>Number of Trades</td>
<td>(NT_i)</td>
<td>-.00</td>
<td>-3.0</td>
</tr>
<tr>
<td>Trading Range</td>
<td>(TR_i)</td>
<td>.28</td>
<td>2.1*</td>
</tr>
<tr>
<td>Appraisal Dummy</td>
<td>(APP_i)</td>
<td>.07</td>
<td>2.4*</td>
</tr>
<tr>
<td>Model Adjusted (R^2) (%)</td>
<td></td>
<td>83.6</td>
<td>71.2</td>
</tr>
</tbody>
</table>

Discount to appraised value is calculated using the average of high and low trading prices less appraised value per unit divided by appraised value per unit. Discounts are expressed as positive values. Log Discount is the minus natural log of the ratio of appraised value per unit to the average of the high and low trading price. Log Price is the natural log of high and low trading price. Log NAV is the natural log of the appraised value per unit. Current yield is the annualized regular cash distribution rate of the partnership per unit divided by appraised value per unit. Leverage is the ratio of secured debt to property at cost. Revenue (earnings) performance is measured as gross revenues (operating surplus) divided by property at cost. Unrealized capital gains are estimates of capital gains taxes that would be incurred if the property was liquidated scaled by the appraised value per unit. Number of trades represent the number of partnership units traded from 12/1/92 through 1/31/93. Trading range is the ratio of the difference between high and low trading price to the average of the high and low trading price. \(APP_i\) is a dummy variable that takes a value of one if the appraisal was performed by an independent third party. The regression models are estimated based on data for 60 real estate limited partnerships as of December, 1992. See Table 1 and the data and methods section for a complete description of the data used to estimated the regression model.

* Significant at the 5% level, two-tailed test.
** Significant at the 1% level, two-tailed test.
mean current yield of 5.2% (see Table 1) to 6.2% is associated with an average change in the partnership discount from 45.4% to 39.5%. These results are consistent with the hypothesis that general partners can improve their reputation by maintaining high current yields on their limited partnership units.

Leverage has a less dramatic impact on the level of the observed discount. For example, on average, a partnership with 20% secured debt would have a percentage discount that is 400 basis points lower than a partnership with no secured debt. This result is consistent with the hypothesis that debt reduces agency costs by tying up free cash flows and increasing the monitoring of the general partnership. This conclusion, however, must be interpreted with caution. Though the parameter estimate on leverage is consistently negative across all estimated models, the statistical significance of the estimated coefficient is sensitive to the model specification.

The trading range, a measure of RELP liquidity, is consistently positively related to the level of the observed discount. RELPs with wide trading ranges trade at a larger discount. For example, on average, increasing a partnership’s trading range by 10% (say from the mean trading range of 13.8% to 23.8%) corresponds with an average increase in the percentage discount of 280 basis points. Interpreting the trading range as a proxy for liquidity indicates that less liquid RELPs trade at a larger discount to their appraised value.

Somewhat surprisingly, percentage discounts are approximately 700 basis points higher for firms that are appraised by an independent third party, as opposed to the general partner. This result, however, is sensitive to model specification. Nonetheless, these weak results suggest that either: (1) when general partners conduct their own appraisals they are conservative relative to independent appraisers; or (2) market prices of RELPs are systematically lower for properties that are appraised by independent third parties.

None of the remaining variables are important determinants of discounts in the sample studied. Neither revenue nor earnings performance are significantly related to the level of the discount. Unrealized capital gains, which were hypothesized to be positively related to discounts, were also empirically unimportant in the determination of the level of the discount. Finally, the number of partnership units traded does not have a significant impact on the level of the discount.

To further test the sensitivity of the results, the estimation sample was separated into two subsamples—one contained the 30 RELPs which were
appraised by the general partner and the second contained the 30 RELPs appraised by an independent third party. The three models (percentage discount, log discount and log price) were separately estimated on each of the subsamples after excluding the dummy variable for appraisal type. Based on these results (not reported), there was no significant difference in coefficient estimates across the two subsamples, with the associated Chow test $F$-statistics (8, 44 degrees of freedom) of 1.53 (for the percentage discount model), .77 (for the log discount model) and 1.24 (for the log price model).

The results provide support for the agency costs and liquidity cost explanations of the observed discounts of RELPs. Consistent with the agency cost hypothesis, RELPs with high current yields and, to a lesser extent, high leverage have lower discounts of market prices from appraised values. Consistent with the liquidity hypothesis, RELPs which trade in a narrower trading range have a lower discount. There is no support for the hypothesis that discounts are related to unrealized capital gains taxes.

**Forecasting Discounts**

The predicted level of the discount ($\hat{D}_i$) for the holdout sample consisting of 41 RELPs was estimated using a parsimonious version of the percentage discount model from Table 3. The percentage discount model was estimated after excluding the four variables (earnings performance, revenue performance, unrealized capital gains, and number of trades) that did not appear as statistically significant in any of the three models. A $F$-test confirms that these four variables are jointly insignificant in all three models, with $F$-statistics (4, 51 or 50 degrees of freedom) of .97, .78 and .75 for the three models. The parsimonious version of the percentage discount model yields the following equation for estimating the predicted discount ($\hat{D}_i$):

$$\hat{D}_i = .73 + (-5.92) CY_i + (-.23)L_i + (.26)TR_i + (.08)APP_i.$$  \hfill (5)

The forecasted discounts are plotted against the observed discounts for the partnerships in Figure 2. If the model provides perfect forecasts of the partnership discounts observed in the holdout sample, all of the points in the figure would lie on the 45 degree solid line. Inspection of this figure suggests that the model performs reasonably well in forecasting the level of the observed discount of RELPs. In addition, the average predicted discount (46.3%) is not significantly different from the average observed discount (44.2%) (a formal test of equality of means yields a $t$-Statistic of .6).
Figure 2 ■ Predicted and observed discount of market price from appraised value for holdout sample of 41 RELPs, December, 1992.

Predicted discounts are based on a parsimonious version of the percentage discount regression model presented in Table 3. Observed discounts are from the January/February 1993 issue of The Perspective.

To more formally test the model’s ability to predict out-of-sample, the observed discount is regressed on the predicted discount:

\[ D_i = d_0 + d_1 \hat{D}_i + u_i. \]  \hspace{1cm} (6)

If the forecasts are unbiased, \( d_0 \) would be equal to zero and \( d_1 \) would be equal to one. The results of the estimation of this regression are presented in Table 4. The intercept term \( (d_0) \) and the slope coefficient \( (d_1) \) are not significantly different from zero and one, respectively. Thus, the regression model yields out-of-sample forecasts that are not significantly biased.

Finally, the forecasts based on the regression model are compared to two alternative forecasting methods. First, assume that limited partnerships sell at their assessed value and thus the mean level of discount is zero. Though this may seem naive, this is essentially the stand taken by the tax court in the Northern Trust Company case (The Northern Trust Company Co., 87 T.C. 349, 1986). In that case the court disregarded an appraiser’s estimated discount because the discount estimate was based on the average discount observed in a particular study. Second, the mean discount observed in the estimation sample of 60 limited partnerships (45.4%) was used. This is essentially the approach taken by the appraiser in the Northern Trust
Table 4 ■ Estimated regression model of observed discount ($D_o$) on predicted discount ($\hat{D}_i$) for holdout sample of 41 RELPs.

<table>
<thead>
<tr>
<th>Variable Description</th>
<th>Symbol</th>
<th>Coef</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td></td>
<td>-.04</td>
<td>-.8</td>
</tr>
<tr>
<td>Predicted Discount</td>
<td>$\hat{D}_i$</td>
<td>1.10</td>
<td>.8</td>
</tr>
</tbody>
</table>

Model Adjusted $R^2$ 67.1%
Model F-Statistic 82.6

Predicted discounts ($\hat{D}_i$) are based on a reduced form of the percentage discount model from Table 3 and is discussed in detail in the text. The regression model is estimated based on data for 41 real estate limited partnerships as of December, 1992.

The $t$-Statistics represent tests of the null hypothesis that the intercept term is equal to zero and the slope term is equal to one.

Company case. Each of the three forecasting methods are assessed on the basis of their root mean squared error. The root mean squared error for each method is estimated as the square root of the average of the squared error terms, where the error term is defined as the predicted discount less the observed discount. The results are presented in Table 5.

This analysis was quite revealing. First, assuming no discount provides the worst forecasts, based on the root mean squared error criterion. Second,

Table 5 ■ Root mean squared errors for holdout sample based on three methods of forecasting discounts of RELPs.

<table>
<thead>
<tr>
<th>Method</th>
<th>Root Mean Squared Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Discount</td>
<td>.51</td>
</tr>
<tr>
<td>Mean Discount</td>
<td>.21</td>
</tr>
<tr>
<td>Regression-Based Discount</td>
<td>.13</td>
</tr>
</tbody>
</table>

Root mean squared errors are calculated as the square root of the average of the sum of squared errors for each method, where errors are defined as the predicted discount less observed discount. The three methods used to predict the discount were:
1. Zero Discount—all discounts were predicted to be zero.
2. Mean Discount—all discounts were predicted to be 45.4%, the Mean Discount for the estimation sample of 60 partnerships.
3. Regression-Based Discount—each discount was predicted based on the reduced form of the percentage discount model of Table 3.
using the mean discount observed in the estimation sample provides considerable forecasting improvement over assuming no discount. Finally, the regression-based forecasts provide, by a large margin, the best forecasts.

Conclusion

The purpose of this research was to provide appraisers with some guidance in the estimation of RELP discounts. An empirical analysis of 112 RELPs revealed that, on average, RELPs trade at a 45% discount from their appraised values. A cross-sectional analysis of the RELP discounts reveals that discounts of market prices from appraised values are lower for partnerships with high current yields, narrow trading ranges for their market prices, and, to a lesser extent, high debt levels.

The relation between current yields, leverage and discounts supports an agency cost explanation for the observed discounts. Reputation is particularly critical in the market for limited partnerships, which are traded in an unorganized secondary market and have been consistently plagued by negative coverage in the financial press. One means of credibly signaling to limited partners the viability of a partnership and the integrity of the general partners is to consistently pay out earnings to limited partners. In addition, partnerships that employ debt have an added mechanism for monitoring the actions of the general partner—the scrutiny of creditors. It is this added scrutiny that leads to the lower observed discounts for the highly-levered partnerships. These results are consistent with the predictions of Jensen (1986) that firms can use dividend and debt policy to alleviate agency costs and increase firm value. In addition, these results complement recent research which indicates that general partners with established reputations are able to garner higher front-end compensation when raising capital for RELP investments (Hamill 1993).

The relation between trading ranges and discounts supports the hypothesis that the lack of liquidity for RELPs leads to discounts in market prices from appraised values. This result is consistent with the predictions of Amihud and Mendelson (1986), that increasing the liquidity of an asset can increase its market value. One implication of this result is that the development of a more organized trading mechanisms for RELPs would lead to a reduction in the observed discounts.

Finally, and perhaps most importantly, the statistical model is able to forecast limited partnership discounts quite well. In a holdout sample of 41 RELPs, compared to applying either a mean discount, or assuming no discount, the regression-based model provides superior forecasts of the discount of market
prices from appraised values. These results suggest that tax courts, which have been reticent to accept the application of a discount from appraised values for RELPs, should view objectively based forecasts such as those developed here favorably.

This research has benefited from the comments of D. Capozza, R. Farrell, C. Liu, D. Rocke, K. Rodriguez, R. Wallace, J. Williams and two anonymous referees. Schultz, Wallace & O’Neil, an accountancy corporation based in Sacramento, CA, was gracious enough to provide the data used in this research. All errors are my own.

References


