

Uphill or Downhill? Locating Your Firm on a Profit Function

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Abstract

Companies know the amount of profit they make, but may not know whether they are operating on the uphill side or downhill side of the profit function with respect to each marketing investment variable. We first identify the risks of an uphill-located firm acting as if it were a downhill-located firm. Then, we develop a diagnostic tool to infer a company's type based on its location on the profit function and to offer guidance for driving it to the neighborhood of the maximum profit. Next, applying the proposed approach to daily newspaper industry data, we analyze the allocation behavior of companies with respect to three marketing efforts: investments in quality, distribution, and advertising sales effort. We find that a majority of the companies are located near the optimal level of spending for quality, which is a surprising finding given that previous studies have characterized marketing managers as over-spenders. In fact, when managers are sub-optimal, they are much more likely to be *under-spending* than over-spending. In other words, newspaper companies are clustered on the uphill side of the profit function. In addition, we provide sales elasticity estimates with respect to quality, personal selling effort and distribution investments, which are sparsely available in the extant literature (compared to price and media advertising elasticities). Finally, we furnish normative and empirical analyses of marketing resource allocation in a dual revenues market with possibly interrelated demands for subscriptions and advertising space.

Managers continually strive to enhance their company's profit. Economic theory advocates that they should operate at the "sweet spot" of a profit function, which is like a bell-shaped curve. For example, in the newspaper industry, managers can increase profit by improving news quality up to a point beyond which further quality improvements fail to attract enough new readers to justify incremental costs. Figure 1 illustrates this reasoning and identifies the sweet spot (q^*) that maximizes a profit function. On the left of this sweet spot, quality reduction reduces profit; on its right, quality enhancement erodes profit. The real problem, however, is that companies

“... are being managed *as though they were on the right or downhill side* of the curve. *In fact, we believe, they are clustered on the left, or uphill side*, where degrading quality creates an imminent danger” (Meyer and Kim 2005, p. 7; emphasis ours).

As described below, the lack of knowledge about the firm's location — uphill or downhill — leads managers to make serious errors in their investment decisions.

Insert Figure 1 about here

Suppose a firm located on the **u**phill side of the profit function (i.e., a type-U firm), believes incorrectly that it is located on the **d**ownhill (i.e., a type-D firm) that generates the *same* amount of profit (see Figure 1). Then, it will disinvest in quality and thereby earn a lower profit (because it is really a type-U firm) instead of increased profit as hoped for. This reduced profit could initiate what Rosenstiel and Mitchell (2004) call the *suicide spiral*, where disinvesting in newspapers leads to circulation declines which leads to revenue declines which leads to more disinvestments, more circulation declines and finally more revenue loss. Similarly, a type-D firm, mistakenly believing that it is a type-U firm making the same profit, would invest in quality and thus decrease its profit. Consequently, it is crucial for managers to know whether their

company is type-U or type-D with respect to a marketing effort before they can implement an appropriate course of action. Hence, a key purpose of this paper is to develop a diagnostic tool to infer a company's type via a valid approach for statistical inference.

Previous research in marketing has investigated issues about measuring market impact (Rust, Ambler, Carpenter, Kumar and Srivastava 2004) and, in particular, under- or over-spending on advertising (e.g., Aaker and Carman 1982, Prasad and Sen 1999). A company is overspending (underspending) when the actual amount exceeds (is below) the optimal amount indicated by a model. However, the optimal amount itself is based on market response elasticity, which is estimated with a measure of uncertainty. How then should a manager determine whether s/he is currently at the sweet spot, and if not, overspending or underspending? Further, how much of a deviation from the sweet spot should s/he tolerate before taking corrective action? To answer these questions, managers need a diagnostic tool to distinguish whether their specific firm is operating in the neighborhood of the sweet spot (i.e., type-N firm) or is it significantly underspending (type-U) or overspending (type-D) — see Figure 1.

The challenge of determining whether or not a given company is a type-N firm — and not either type-U or type-D — is to quantify an interval (or a region), say (\underline{x}, \bar{x}) around the sweet spot x^* within which the firm is considered close to the optimal, where $\underline{x} \leq x^* \leq \bar{x}$. Previous marketing studies concentrated on determining the response elasticities and/or the optimal x^* and have not addressed this problem of estimating the triplet $(\underline{x}, x^*, \bar{x})$, e.g., see the literature reviews by Gatignon (1993) and Mantrala (2002) and research on under- and over-spending behaviors (e.g., Mantrala, Sinha, Zoltners 1992, Joseph and Richardson 2002, Naik and Raman 2003). Econometric theory, however, provides a few avenues to solve this inferential problem. One approach, based on the delta method (e.g., Davidson and MacKinnon 2004, p.

202), determines the interval (\underline{x}, \bar{x}) that covers a neighborhood of x^* with 95% probability, which is obtained via an approximate normal distribution. An alternative approach, proposed by Krinsky and Robb (1986), avoids approximations by using Monte Carlo samples for constructing 95% coverage probability empirically. We follow the latter approach in this paper to develop a diagnostic tool for locating each firm on the profit function and thus providing *firm-specific* diagnostic inferences. Based on actual spending decisions, we illustrate how a given company can infer whether it is type U (below \underline{x}), type N (between \underline{x} and \bar{x}), or type D (above \bar{x}) with respect to multiple marketing variables. This knowledge of a firm's type (i.e., uphill or downhill or neither), as discussed earlier, is crucial for implementing an appropriate course of action to enhance profitability.

The proposed approach yields new substantive findings. Specifically, utilizing data from hundreds of daily newspaper firms collected by the Inland Press Association (Inland) over four years (1998 through 2001), we investigate the optimality of these firms' investments in three marketing variables: quality, distribution, and ad space selling effort. Interestingly, we find that over 70% are type-N firms, i.e., a majority of these companies are located near the sweet spots of all three investments. This result is surprising because previous studies have characterized marketing managers as typically overspenders (e.g., Aaker and Carman 1982, Prasad and Sen 1999). Further, we find that when firms are sub-optimal in their quality and advertising sales investments, they are much more likely to be underspending than overspending. This finding corroborates Meyer and Kim's (2005, p. 7) view that newspaper companies are "clustered on the uphill" of a profit function.

In addition to the above two contributions (diagnostic tool and new empirical findings), this article further augments the marketing literature in two important ways. First, we provide

elasticity estimates for quality, personal selling effort and distribution investments which are sparsely available in the extant literature (e.g., Hanssens, Parsons, Schultz 2001, pp 347-349) compared to estimates of advertising or price elasticities (e.g., Assmus, Farley and Lehmann 1984, Tellis 1988). Specifically, we find that on average, newspaper distribution elasticity is 0.23, advertising sales effort elasticity is 0.54 and news quality elasticity is 0.49. This last finding should assure publishers that “good news quality is good business” (Overholser 2004).

Second, the newspaper industry offers a novel marketing context due to its *dual revenues* market structure: sales of news to the readers and access to specialized audiences to the advertisers (Picard 1994). Furthermore, the resulting two revenue sources can be interrelated. That is, the desirability of a newspaper to advertisers and, hence, the demand for advertising space increases as circulation increases; in turn, advertising volume carried by a newspaper can influence its subscription sales (e.g., Blair and Romano 1993, Picard 1994, Depken and Wilson, 2004). So far, there appears to be no research on optimal investments and allocation of marketing resources for dual markets with such demand interdependency (see Mantrala 2002, Gatignon 1993), which further motivates our normative and econometric analyses.

The rest of this article is organized as follows. In the next section, we review relevant literature pertaining to the newspaper industry’s market structure and marketing efforts. Using this knowledge, we then formulate and analyze an interrelated demands model and deduce normative implications for managing dual revenues markets. Next, we specify and estimate an econometric model, develop the diagnostic tool to infer the location of a firm on the profit function, and present the empirical results based on the Inland data. Finally, we conclude by summarizing the managerial implications.

Literature Review

We first describe the market structure within the newspaper industry and then review the communications literature for extant findings on marketing-mix effects.

Market Structure

Historically, the newspaper industry has been one of the most profitable of all industries with operating profits averaging about 15% and net profits averaging about 12% in 2001 (Picard 1994). In recent years, however, public newspaper companies are being challenged by declining circulations; for example, paid subscriptions dropped from 353 per 1000 population in 1950 to 202 per 1000 in 2000. Other challenges include printing capacity constraints, price ceilings, and competition from the Internet for classified advertising revenues. The two main characteristics of the newspaper industry are its monopoly structure and demand interrelatedness.

Monopoly. This structure is the rule rather than the exception in the US daily newspaper industry. Specifically, 98% of newspapers exist as the only daily paper published within their local markets today (Blair and Romano 1993; Bucklin, Caves and Lo 1989; Picard 1994). An explanation for the emergence of monopoly is the *circulation-advertising death spiral*: In markets where two or more daily newspapers compete, the newspaper with the largest circulation and highest market penetration attracts a disproportionate amount of advertising — even when circulation differences are small — while secondary papers enter into a circulation death spiral of lower advertising revenues leading to disinvestments leading to lower circulation leading to lower advertising and so on, ultimately eliminating one of them from the market (Picard 1994). In the few cities where limited competition exists, it nearly always occurs between highly differentiated newspapers such as a broadsheet and a tabloid intended for different audiences (Picard and Brody 1997).

Demand interrelatedness. As subscription sales increases, demand for advertising space increases because the advertising cost per thousand readers reached decreases. In addition, advertisers are attracted to newspapers with large paid circulations because subscribers who pay for the newspapers are more likely to read and register their advertised messages. In turn, as already mentioned, advertising volume in a newspaper can impact its circulation sales (Bucklin et al. 1989; Corden 1953, Ferguson 1983; Rosse 1967). One school of thought suggests that this impact is positive because advertising provides information on the availability of products and their prices; this information is considered valuable by many consumers, and it can be skipped over by those who do not find it valuable (Blair and Romano 1993). Consequently, the demand for a newspaper can increase as advertising volume in it increases. On the other hand, advertising may negatively impact circulation sales because it lowers the value of the newspaper to readers who seek to be informed of news and not commercial matters (Abrahamson 1998). Such advertisement may vary across cultures. For example, Sonnac (2000) provides some empirical evidence that readers' attitudes toward press advertising are country-specific. Whereas American readers are ad-lovers, European readership seems in majority to be ad-averse. Our research allows for and investigates both scenarios of interrelated demands.

Marketing Mix Effects

Three major marketing-related expenditures of daily newspapers are related to (1) enhancing *news quality* managed by the Editorial Department, which consists of news reporters, section editors, copy editors, photographers, graphic artists, and administrative staff; (2) growing the *distribution coverage*, a responsibility of the Circulation-Distribution Department, which employs telemarketers, newspaper distributors, paperboys, motor carriers, delivery truck operators, and coin-box rack technicians; and (3) generating advertiser revenues managed by the

Advertising Department, which uses an ad space sales force with support from creative design artists and client service personnel. A brief review of past research with respect to these three newspaper marketing efforts follows.

News Quality. The Editorial Department focuses on the upkeep of journalistic quality, editorial independence and integrity. A persistent question asked in the newspaper industry is, “Is good news quality good business?” It is only recently that expenditures on news quality have begun to be viewed as a marketing investment rather than an operating cost. Litman and Bridges (1986) put forth the concept of “financial commitment” that suggests newsroom investment as a surrogate measure of news quality. Rosenstiel and Mitchell (2004) elucidate the concept of the suicide spiral that describes how disinvestments in quality cascade into declining subscriptions. Empirically, Cho, Thorson and Lacy (2004) show that newsroom expenditures are positively correlated with quality news content, which is positively correlated with higher circulation sales. This stream of research, however, stops short of providing firms with guidance on determining the optimal investment in quality.

Circulation-Distribution. Distribution investments include compensation of telemarketers and delivery agents plus expenditures on delivery systems (e.g., data processing, postage, repairs, wire services). The Newspaper Association of America (NAA) has urged newspapers to view their delivery systems as strategic assets and their overall distribution investments as a means to increase sales and profit (NAA Task Force Report, 2000). For example, newspapers like the *Sacramento Bee* increased their subscriptions and profits by developing superior delivery systems (Stein 1995).

Advertising Space Sales. The expenditures on this effort are primarily related to sales representatives selling ad space to local retailers and businesses. Smith (1998) shows that

customer traffic (i.e., subscriptions, customer count), news content, and salesperson's effectiveness influence firms to buy ad space in a newspaper. In a later study, Smith (2001) shows that sales efforts by newspapers tend to increase advertising revenue, which, in turn, can lead to more subscriptions.

We close this review by noting the constancy of price and margins and the limited research on optimal resource allocation in the newspaper industry.

Price, Margins and Resource Allocation. Daily newspaper retail prices remain unchanged over spans of four to seven years (e.g., Bils and Klenow 2002). With regard to price of advertising space, most newspapers publish in advance their rates for retail and classified advertising of various sizes and formats, which are held fixed for at least a year. In addition, newspaper's gross margins on each circulation dollar and on each ad revenue dollar are constant because, after the first copy of each day's newspaper is produced, variable costs, primarily newsprint costs, associated with printing of subsequent copies remain constant.

Finally, the extant literature in the fields of Journalism and Economics is descriptive with limited attention to normative analyses of marketing decisions. Two notable exceptions are Corden (1953) and Bucklin, Caves and Lo (1989), who considered maximization of a newspaper company's profit with respect to prices, product quality and circulation-enhancing investments. However, these studies did not incorporate (i) the role of inter-related dual revenue sources as a function of investments in quality, distribution and advertising sales effort nor provide (ii) firm-specific inferences on its location on the profit function (i.e., "How far are we from the optimal?"). Hence, we next formulate a dual revenue market model to investigate these issues of marketing resource allocation and inference.

Model Formulation and Normative Analyses

The two revenues sources of a newspaper company are paid circulations and advertising revenue. Let S denote the number of subscribers for the year, m_1 = margin (\$) per issue (price minus cost), and k = number of issues per year. Typically, k equals the number of days in the year; so $k = 730$ when some newspapers produce both morning and evening editions. Let R denote revenues from all advertiser sources (e.g., retail and classified advertising), m_2 denote the margin per advertising dollar generated. Based on the literature review, subscription sales S and advertising revenue R depend on investments made in news quality, distribution coverage and advertising selling efforts. Denoting (q, d, a) as dollars invested in quality, distribution, and advertising sales, respectively, we can express the dual revenue sources as

$$(1) \quad S = f_1(q, d, R) \quad \text{and}$$

$$(2) \quad R = f_2(a, S),$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are general response functions that increase at a decreasing rate with respect to each argument (i.e., $\partial f_i / \partial x > 0$, $\partial^2 f_i / \partial x^2 \leq 0$, $i = 1, 2$, and $x = q, d, a, R, S$ as necessary). Note the allowance of interrelatedness of demand: advertising revenue R influences subscriptions in equation (1), and subscriptions stimulate advertising revenues in equation (2).

Summing the two revenues and deducting the expenditures on quality, distribution and advertising sales, we obtain the firm's profit as follows:

$$(3) \quad \pi(q, d, a) = m_1 \times k \times S + m_2 \times R - q - d - a.$$

To determine the optimal investment levels in quality (q^*), distribution (d^*), and advertising sales (a^*), we differentiate equation (3) with respect to (q, d, a) and solve the resulting three first-order conditions simultaneously. We relegate those details to the Appendix and describe below the

analytical results presented in Table 1, focusing on appropriate decision rules for the various special cases or *types* of dual revenue markets.

Specifically, we identify *four types* of markets based on the cross-market dependency coefficient $\delta = 1 - \frac{\partial f_1}{\partial R} \frac{\partial f_2}{\partial S}$, which arises in equation (A4) of Appendix. The presence of this dependency coefficient makes the optimality conditions for dual revenue markets different from those in the standard analysis (Dorfman and Steiner 1954; see Hanssens, Parsons and Schultz 2001, p. 358-361). The four types of markets are as follows.

Market Type 1: Unrelated markets. In this special case, the two revenue sources are not related, i.e., we have $\partial f_1/\partial R = 0$ and $\partial f_2/\partial S = 0$, and so the dependency coefficient equals unity (i.e., $\delta = 1$). Although this market is valid for other industries and analyzed by Dorfman and Steiner (1954), it is unlikely to prevail in the daily newspaper industry.

Market Type 2: Partially related markets. Here subscription increases advertising revenues, but not vice versa (i.e., $\partial f_2/\partial S > 0$ and $\partial f_1/\partial R = 0$). This market setting is analyzed by Corden (1953) and Bucklin et al. (1989), and it differs from this study because we identify and analyze interrelated demand markets as described below.

Market Type 3: Interrelated markets with opposing feedback effects. This market arises when increased circulation begets large advertising revenues, but the resulting enhanced advertising volume *reduces* the subscription sales, thereby implying negative feedback from advertising revenues to subscriptions, i.e., $\partial f_2/\partial S > 0$ and $\partial f_1/\partial R < 0$. Such a market is suggested by Abrahamson (1998), Sonnac (2000), based on the argument that increasing volumes of advertising cause reader irritation and erode the utility a subscriber gets from the newspaper. We will empirically test the implied hypothesis $H_0: \delta > 1$.

Market Type 4: Interrelated markets with positive feedback effects. Rosse (1967), Ferguson (1983), and Blair and Romano (1993) support the presence of such markets, where both feedback effects are positive (i.e., $\partial f_1/\partial R > 0$ and $\partial f_2/\partial S > 0$). Empirically, to distinguish this possibility from Type 3 markets, we need to test whether or not $H_0: \delta < 1$ holds.

 Insert Table 1 about here

The various market types induce *different* investment behaviors from managers. Table 1 summarizes the optimal investments in quality q_j^* , distribution d_j^* , and advertising sales a_j^* , where j indicates the market type ($j = 1, 2, 3$, or 4). For markets of type 1, we recover the usual optimality conditions stated in Dorfman-Steiner's (1954) theorem. In markets of type 2, because of positive feedback from subscriptions to ad revenues, the optimal investments in quality and distribution are larger than those for unrelated markets (i.e., $q_2^* > q_1^*$ and $d_2^* > d_1^*$), while the optimal advertising sales expenditure remains the same ($a_2^* = a_1^*$). Next, for market type 3, managers should *reduce* the investments in news quality, distribution and advertising sales relative to those made for partially related markets (i.e., $q_3^* < q_2^*$; $d_3^* < d_2^*$; $a_3^* < a_2^*$). Intuitively, increasing the investments is counterproductive because advertising revenues hurt subscriptions in this market. When we compare market types 3 and 1, we find again that managers in market type 3 should *invest less* in news quality and distribution than the corresponding levels for market type 1 (i.e., $q_3^* < q_1^*$, $d_3^* < d_1^*$), except when the margin ratio exceeds the feedback effect (i.e., $m_2/km_1 > |\partial f_1/\partial R|$). In other words, investments in quality and distribution greater than the optimal levels for market type 1 are justified in market type 3 *only if* profit contributed by the increased advertising revenue exceeds the lost contribution due to lower subscriptions. In

contrast, when we compare market types 4 and 1, we observe that managers in market type 4 should *increase* the investments in quality, distribution and advertising sales beyond the corresponding levels for market type 1 because positive feedback reinforces *both* subscription and advertising revenues.

In sum, the above normative analysis reveals four market types and appropriate decisions for each one. Furthermore, previous research pertains only to market types 1 and 2 (see Bucklin et al. 1989; Corden 1953; Dorfman and Steiner 1954). Consequently, we identified new markets (namely, market types 3 and 4), derived the optimal investments in quality, distribution, and advertising, and gained insights into *different* decision rules applicable across these markets. Interestingly, investment decisions in market type 3 tend to be opposite of those for market type 4 (see Table 1 for details). Hence, managers need to know the type of market they operate in, an empirical issue that is examined in the next section.

Empirical Analyses and Diagnostic Tool

In this section, we present empirical analyses of Inland daily newspaper data that allows us to develop and demonstrate two diagnostic procedures: (1) an approach for managers to determine the type of market (types 1, 2, 3 or 4 defined above) in which they operate; and (2) a tool to assess whether their firm is over-spending or under-spending and offer guidance for driving it to the neighborhood of the maximum profit.

Determination of market type depends on the estimation of the cross-market dependency coefficient. To this end, we first describe the datasets, model specification and estimation approach, and then present the empirical results for cross-market dependency and elasticity estimates for advertising sales, distribution and quality. Subsequently, we propose the diagnostic

tool, which is based on a 5-step algorithm, to locate a specific company on the multivariable profit function and thus infer whether it is a firm of Type-N (near-optimal spending), Type-U (under-spending) or Type-D (over-spending) with respect to each of the marketing investments.

Inland Data

Inland Daily Press Association constructs annual datasets that “tell us more about the economic innards of American dailies than any other source” Blankenburg (1989, p. 98). Inland datasets contain information from yearly samples of hundreds of individual newspapers with circulations not exceeding 85,000, comprising a representative cross-section of the total 1,400 US daily newspapers (Picard 1989, p. 110). Our research utilizes Inland data collected in the years 1998, 1999, 2000, and 2001. Because newspaper companies desire confidentiality, Inland datasets do not reveal information on their identities and locations. Each year’s dataset contains information on participating newspaper’s subscription sales (S), advertising revenues (R), margins on sales and ad revenues (m_1, m_2). In addition, we have data on the firms’ annual investments in news quality, circulation-distribution and advertising sales activities.

Specifically, news quality investment (q) equals the sum of news-edit expenses, newsroom salaries, and miscellaneous expenses; distribution investment (d) equals the sum of distribution expenses, distribution department salaries, and miscellaneous expenses; advertising sales investment (a) equals the sum of expenses to reach potential advertisers, ad department salaries, and miscellaneous expenses.

Response Model Specification

Consistent with the theoretical model expressed by (1) and (2), we specify the two-equation simultaneous system of a newspaper company’s dual demands as follows:

$$(4) \quad S_k = \alpha_0 + \alpha_1 R_k + \alpha_2 \text{Ln}(q_k) + \alpha_3 \text{Ln}(d_k) + \varepsilon_k, \text{ and}$$

$$(5) \quad R_k = \beta_0 + \beta_1 S_k + \beta_2 \text{Ln}(a_k) + v_k,$$

where S_k denotes annual subscription sales (in thousands), R_k represents ad revenues (in hundred thousand dollars), $\text{Ln}(\cdot)$ is the natural logarithm, and (q_k, d_k, a_k) are dollars invested in quality, distribution and advertising sales, respectively, by the newspaper companies $k = 1, \dots, K$. The response coefficients for the quality investment coefficient α_2 , the circulation-distribution investment coefficient α_3 , and the advertising sales investment coefficient β_2 are all expected to be positive. Also, we expect the impact of subscription sales on ad revenues β_1 to be positive, whereas the impact of ad revenues on subscriptions α_1 may be zero, positive, or possibly negative. To sign the intercept term β_0 in Equation (5), we note that advertisers will not advertise in a newspaper unless it has some minimum positive subscriptions. Therefore, if the newspaper's advertising effort is negligible (say close to \$1), subscription sales must be greater than $(-\beta_0/\beta_1) > 0$ for advertising revenues to be positive. Consequently, given $\beta_1 > 0$, the intercept $\beta_0 < 0$.

Turning to Equation (4), note that a minimum investment in news quality (editorial or newsroom employees) is necessary to avoid closing the newspaper. Assuming this minimum investment in quality, which will provide for some minimum subscription sales, the intercept α_0 in Equation (4) implicitly captures the incremental impact of the annual carryover effect of previous distribution investments.

Lastly, the random errors, ε_k and v_k , are normally distributed with zero means, constant variances $(\sigma_\varepsilon^2, \sigma_v^2)$, and possibly correlated $(\rho_{\varepsilon v})$ because common economic conditions affect both subscriptions and ad revenues. More importantly, these specifications satisfy all the

theoretical assumptions: it can be verified via (4) and (5) that $\partial E[S_k]/\partial z > 0$, $\partial^2 E[S_k]/\partial z^2 \leq 0$, $\partial E[R_k]/\partial z > 0$, and $\partial^2 E[R_k]/\partial z^2 \leq 0$ for every firm k and every argument $z = q_k, d_k, a_k, R_k$, and S_k .

Accounting for heterogeneity in model parameters across newspapers

Equations 4 and 5 assume the model parameters are identical across newspapers, which may not be a realistic assumption when we consider heterogeneity due to location, population, economic development, diversity, and retail business concentrations across our annual, cross-sectional samples of newspapers. For example, subscriptions and ad revenues of newspapers located in smaller cities can be systematically more or less responsive to quality investment efforts than those of newspapers located in larger city markets. To incorporate heterogeneity, we partition the annual Inland samples into homogenous subgroups or segments using latent class clustering while, alternatively, a random coefficients model (Swamy 1970) could be estimated (e.g., Rao, Agarwal and Dalhoff 2004).

Although the Inland datasets do not provide any direct information on the identities and geographic locations/markets of newspapers, they do contain data on several variables: Total Salaries of all employees; Number of Newsroom Employees; Number of Circulation-Distribution employees; and Number of Advertising Department Employees. As we explain below, these variables not only reflect firms' sizes, but also serve as indicators of the categorical latent variable for classifying newspapers into groups with similar geographic characteristics.

Total Salaries (TOTSAL): In the US, significant variation exists in the wages paid for particular jobs in different geographic locations and regions (see, e.g., Mercer Human Resources Consulting's 2005 *Geographic Salary Differentials* study at imercer.com). Such geographic variation in salaries reflects heterogeneity in the cost of labor (i.e., differences between localities in terms of cash compensation for the same work) and the cost of living (i.e., difference between

localities in terms of cost of housing, groceries, transportation, and entertainment). Further, market pay for lower-paid and lower-level employees varies more with geography than that for higher paid and more senior employees because employers generally recruit for lower paying jobs from the local population. In the case of newspapers, the Readership Institute at Northwestern University reports that they recruit over 84% of their employees from outside the newspaper, and the bulk of these external hires are rank-and-file or lower-paying jobs staffed from within the newspaper's community or market (1999 Workforce Characteristics Survey, *readership.org*). This observation and the fact that most daily newspapers have only one primary office location and few or no secondary locations suggest that variations in total salaries across newspapers may reflect differences in the cost-of-labor and cost-of-living in their headquarter towns and environs.

Number of Newsroom Employees (NEMP): According to the 1999 Workforce Characteristics Survey of the Readership Institute, thirty-five percent of the newspapers' employees work in the newsroom (i.e. News-Editorial department). While an industry guideline calls for one newsroom employee slot per 1,000 circulation, larger newsroom staffs exist among papers that produce more *zoned editions*, which offer more specialized coverage for a host of special interest groups including ethnic groups like Hispanics and affluent immigrant groups (Rosentiel and Mitchell 2004). Thus, newspapers in locations with more diversity and/or higher education and income levels are likely to have larger numbers of newsroom employees.

Number of Circulation-Distribution Employees (DEMP): Circulation department employees account for another 35% of a newspaper firm's total employees (1999 Workforce Characteristics Survey, *readership.org*). These employees' ensure the delivery of newspapers to all its subscribers and other outlets in the newspaper's market. Consequently, we expect newspapers

with greater numbers of circulation department employees to be located in more urban areas with larger populations.

Number of Advertising Department Employees (AEMP): On average, about 25% of a newspaper's staff work in the Advertising Department to generate advertising revenues. A large proportion of these constitute the advertising space sales force. We expect newspapers with higher numbers of Advertising Department employees and, therefore, larger advertising sales force sizes to be located in areas with greater numbers of business accounts.

Using these four indicators, we incorporate heterogeneity in model parameters due to observed and unobserved characteristics of newspaper firms and their geographic markets via a three-step latent class segmentation approach. First, we employ latent class clustering (LCC) analysis (e.g., Vermunt and Magidson, 2003) to identify clusters of newspapers with similar characteristics. The LCC model includes a K-category latent variable, measured by the above four indicators, where each category represents a cluster. As the number and composition of latent segments are not known a priori, we investigate different latent class clustering scenarios such as one-segment, two-segment and three-segment solutions. Furthermore, because LCC formulates a finite mixture of multivariate normal distributions, it provides probabilistic classification of newspapers into clusters, does not require rescaling of observed variables, and yields managerially meaningful segments (e.g., Wedel and Kamakura, 2000, p.78 & 329). Second, we estimate the equations (4) and (5) for each segment under various clustering scenarios. In each of these model estimations, we allow for the possibility of correlated errors and employ the three-stage least squares (3SLS) estimation approach that utilizes complete information on both the structural equations. (The Lagrange Multiplier test, e.g., Greene 2000, p. 492, was applied in each estimation scenario to establish that 3SLS estimation was more

efficient than the 2SLS estimation procedure that ignores the correlation between errors.) Finally, we select the best segmentation-estimation model by applying the mixture regression criterion (MRC), which was developed by Naik, Shi and Tsai (2006). The MRC extends the classical Akaike Information Criterion (AIC) to finite mixture regression models by deriving theoretically a clustering penalty term to prevent over-clustering (i.e., inclusion of insignificant segments).

By applying the above approach to Inland data, we find that the MRC achieves its minimum value for the two-segment model in each year (see Table 2). In other words, the two-segment model balances both the fidelity (i.e., superior fit) and parsimony (i.e., fewer parameters) better than one- or three-segment models. Therefore, in the rest of this section, we further examine the two-segment estimation results.

Insert Table 2 about here

Segmentation results

In each of the four years, the Wald statistics associated with the parameters of each of the four indicators in the 2-segment clustering solution were significant at the 95% level of confidence, establishing that they were useful in exposing latent differences between the segments. Further, newspapers assigned to Segment 1 have high probabilities of having smaller observed values on each of the four indicator variables. Table 3 displays the sizes and indicator profiles of the derived segments. We observe that, in each year, Segment 2 consists of newspapers whose average total salaries are about 4 times larger than the corresponding mean total salaries of newspapers in Segment 1. Also, the mean numbers of newsroom, circulation-department and advertising sales employees of Segment 2 newspapers in each year are more than 2.5 times larger than those of Segment 1 newspapers. Hereafter, we shall refer to Segment 2

(Segment 1) as the “Large Firm” (“Small Firm”) segment. On average, 57% of all firms fall in the Small Firm segment.

Insert Table 3 about here

Next, Table 4 reports the means and standard deviations of the three input investment variables, subscription sales, ad revenues, and margins for the two segments in each year. We see that the Large Firm segment’s mean investments in quality, distribution and advertising sales effort are 3.5 to 5 times greater than the corresponding Small Firm segment’s mean investments. Further, in each year, the Large Firm segment’s mean subscription sales are about 3 times greater and advertising revenues over four times greater than those of the corresponding Small Firm segment.

Insert Table 4 about here

Segment-level simultaneous equation model estimation results

Estimates of model parameters. Table 5 reports the segment-by-segment dual revenues response model parameter estimates across the years 1998 through 2001. First, based on R-squared values that range from 0.77 to 0.86, the proposed simultaneous-equation model for subscription and ad revenues fits each segment’s data in each year satisfactorily. Second, across the four years, the estimated slope coefficients of all the variables have the correct signs. Also, across all segments and years, the estimated intercepts (β_0) possess the expected negative sign and remain significant at 90% or greater confidence level.

Insert Table 5 about here

Third, 10 out of the 12 investment coefficient estimates in the Small Firm segment across the 4 years are statistically significant at the 95% confidence level, while 7 out of the 12 estimated investment coefficients in the Large Firm segment are significant at the 90% confidence level. Focusing on the Small Firm segment model estimation results, we observe that the estimated coefficients for investments in quality (α_2) and advertising sales effort (β_2) are statistically significant at the 95% confidence level in all four years while the distribution coefficient estimate (α_3) is not significant in 1998 and 1999 but is significant at the 95% confidence level in 2000 and 2001. The corresponding significance results in the Large Firm segment are mixed. In this segment, none of the investment coefficient estimates are significant in 1998; the quality investment is significant at the 95% confidence level only in year 1999; while the distribution and advertising sales investment coefficients are statistically significant at the 95% confidence level in year 1999, and at the 90% confidence level in years 2000 and 2001. Overall, we find that investments in newspaper quality have consistently significant positive effects in the Small Firm segment, while advertising sales investments tend to have significant positive effects in both Small Firm and Large Firm segments.

Fourth, we note that in both segments and in all four years, subscription sales have positive and statistically significant impacts on advertising revenues and, similarly, advertising revenues have a positive and significant impact on subscription sales except for the Small Firm segment in 1998 when the advertising revenues impact on subscriptions was not significant.

Next, based on these results, we investigate whether the newspaper market type is one of interrelated demands, partially related demands or unrelated demands in the four years under study.

Determining the Market Type. To this end, we test a set of hypotheses hierarchically as follows. First, test whether $\delta > 1$ using one-tailed t-test; if it holds, then the market is type 3. If not, next test $\delta < 1$ using one-tailed t-test; if it holds, then market is type 4. Otherwise $\delta = 1$, and so test $\beta_1 = 0$ using two-tailed t-test. If it holds, market is type 1; else type 2.

To conduct these tests, we need to evaluate the mean $E[\delta]$ and variance $\text{Var}[\delta]$, which are given by $E[\delta] = 1 - \alpha_1\beta_1$ and $\text{Var}[\delta] = (\beta_1 \ \alpha_1)\Sigma_{\alpha\beta}(\beta_1 \ \alpha_1)'$, where $\Sigma_{\alpha\beta}$ denotes the 2×2 variance-covariance matrix of $(\alpha_1 \ \beta_1)'$ estimates. For example, for the 1998 estimates of $(\alpha_1 \ \beta_1)$ in the Large Firm segment given in Table 5, we find that $\hat{\delta} = 0.50$ its variance is 0.056, and so the resulting t-value = -2.11, which indicates that $\delta < 1$ and, hence, this market is type 4 (i.e., interrelated markets with positive feedback). Similar results emerge for the Large Firm segment in all the remaining years as well as in the Small Firm segment with the sole exception of the result in 1998 where the t-value suggests that $\delta = 1$ (see Table 6). Furthermore, in this case, Table 5 indicates that we must reject $\beta_1 = 0$; hence, this market is type 2 (i.e., partially related). Overall, our empirical findings corroborate the markets described by Corden (1953) and Bucklin et al. (1989) and furnish reasonable support for the view that US daily newspapers operate in markets of type 4, characterized by interrelated demands for subscriptions and advertising space with positive feedback effects. This conclusion is also consistent with Sonnac's (2000) observation that newspaper readers in the US tend to be "ad-lovers."

 Insert Tables 6 and 7 about here

We now assess the estimated response elasticities in the two newspaper market segments across the four years.

Newspaper Quality, Distribution and Selling Effort Elasticities

Table 7 reports the overall and segment-level elasticities of both subscription sales and advertising revenues with respect to the three marketing activities in each year. First, the estimates of quality, distribution and personal selling elasticities in the daily newspaper industry are consistent with earlier, albeit sparsely, available estimates in the marketing literature (e.g., Hanssens, Parsons, Schultz 2001). Specifically, averaging across the two segments and four years, we obtain the mean elasticities. The elasticity of subscription sales and advertising revenues with respect to news quality investments are 0.49 and 0.55, respectively, which are consistent with the overall quality elasticity estimates of 0.521 reported by Lambin (1976) and 0.611 reported by Ramaswamy et al. (1993). Next, we obtain mean subscriptions-distribution and advertising-revenues elasticities of 0.23 and 0.26, respectively, which fall in the range of previously reported distribution elasticities for cigarettes (0.18), automobiles (0.49) and VCRS (0.49) (cf. Andrews and Franke 1996). Lastly, the mean elasticity of ad revenues response with respect to ad space-selling effort of 0.54 is consistent with previous estimates of 0.5 (ranges from 0.26 to 0.98) reported by various studies on youth enlistment in military (see, e.g., Hanssens and Levien 1983, Ramaswamy et al.'s 1993 PIMS data-based study). However, the mean elasticity of subscription sales to ad space selling effort is about 0.16, which is lower because this marketing activity exerts an indirect effect.

Second, the results in Table 7 highlight the powerful role of investments in news quality, in influencing subscription sales, which in turn drives advertising revenues. Indeed, the impact of news quality investment on advertising revenues is as large as the impact of ad space selling effort and somewhat stronger than its direct impact on subscription sales. This result is especially true in the Small Firm segment, where the average elasticities of subscriptions and advertising revenues with respect to news quality investments are as high as 0.57 and 0.64

respectively, significantly greater than the corresponding values in the Large Firm segment which are about 0.4 and 0.47. These differences between the two segments appear consistent with industry reports that the smaller newsroom staffs of smaller newspapers are used much more intensively than those of larger newspaper firms (Rosentiel and Mitchell 2001). However the subscriptions-distribution and ad revenues-distribution elasticities in the Large Firm segment are greater than those observed in the Small Firm segment, perhaps because small newspaper markets are closer to saturation with respect to distribution coverage than Large newspaper markets.

Diagnostic tool for assessing under- or over-spending and enhancing profit

Although elasticity estimates are informative, they do not provide sufficient information to decide whether managers should actually increase or decrease investments in those activities from current levels. For example, for any magnitude of quality elasticity (large or small), it is prudent to decrease (increase) investments in quality if the newspaper company is located on the downhill (uphill) of the profit function (see Figure 1). Hence, to make profitable decisions, managers need to know the firm's location on the profit function. To facilitate this goal, we present a diagnostic tool based on the following algorithm:

- Step 1.* For a given company k , derive the optimal decisions (q_k^*, d_k^*, a_k^*) as functions of the model parameters. Let $g_k(\theta)$ be such a 3×1 vector-valued function of the vector of parameters θ .
- Step 2.* Draw one thousand samples $\tilde{\theta}_i$ from the normal distribution $N(\hat{\theta}, \hat{\Sigma})$, where $i = 1, \dots, 1000$. This step accounts for both the magnitude of the estimates $\hat{\theta} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\beta}_1, \hat{\beta}_2)'$ and the uncertainty associated with them via their variance-covariance matrix $\hat{\Sigma}$.
- Step 3.* Evaluate the expressions in step 1 using the realized parameter values in step 2. Let $g_{ki} = g_k(\tilde{\theta}_i)$ be the evaluated quantities for the company k at the realized parameter values in the i^{th} draw.

- Step 4.* Sort the g_{ki} values in ascending order for each activity and locate the 25th and 975th values to obtain the lower and upper confidence limits ($\underline{x}_k, \bar{x}_k$), respectively.
- Step 5.* If the actual expenditure on an activity lies within its corresponding confidence limits ($\underline{x}_k, \bar{x}_k$), the investment decision for that activity is nearly optimal. If the actual expenditure is below \underline{x}_k , it represents under-spending; if above \bar{x}_k , then over-spending.

To derive the expressions required in step 1, we maximize profit in (3) subject to the demand system given in (4) and (5). The resulting optimal decisions are

$$(6) \quad \begin{bmatrix} q_k^* \\ d_k^* \\ a_k^* \end{bmatrix} = \begin{bmatrix} \frac{m_{1k}\alpha_2 + m_{2k}\alpha_2\beta_1}{1 - \alpha_1\beta_1} \\ \frac{m_{1k}\alpha_3 + m_{2k}\alpha_3\beta_1}{1 - \alpha_1\beta_1} \\ \frac{m_{1k}\alpha_1 + m_{2k}\beta_2}{1 - \alpha_1\beta_1} \end{bmatrix} = g_k(\theta).$$

This 5-step algorithm comprises the diagnostic tool, which determines a firm's location on the profit function and enables managers to steer their companies toward optimality. To elucidate this point, consider here a simplified situation where a manager seeks to maximize profit (π) by changing investment levels in quality (Q) and distribution (D). In Figure 2, Panel A locates the Firm A on the profit function $\pi(Q, D)$, and Panel B shows its location in the decision space (Q, D). In practice, the manager does not “see” the firm on a profit function in the manner you see the Firm A in Panel A. Rather, as in Panel B, the manager knows only the invested amounts (q_1, d_1) and the associated profit π_1 . In other words, the manager's business reality is the decision space in Panel B. This decision space becomes three (or more) dimensional when additional decisions are to be made (as in the empirical study). In such cases, without analytic tools, it is humanly impossible to “see” the profit function. Even considering the simplified situation in Panel B, should the manager decrease Q, increase D, or try some combination (say)

that increases both Q and D (see Panel B)? Such decisions move the firm to another location (q_2 , d_2), shown by the dotted circle in Panel B, *which may or may not generate enhanced profit*.

Consequently, how should one assess whether a manager's decisions brought the firm closer to the optimal?

The proposed diagnostic tool provides this crucial information. Panel C illustrates how the diagnostic tool works: (i) it first projects the firm A from its current profit level into the decision space; (ii) then constructs iso-profit contours in the decision space; and (iii) finally identifies the optimal path to achieve the maximum profit. The shaded region in Panel C denotes the confidence region. Although the confidence region provides useful information on over- or under-spending, it does not offer guidance to a manager to drive the firm from its current location into the neighborhood of maximum profit. This guidance comes from the optimal path that traces a perpendicular to iso-profit contours (see the curved arrows), which the diagnostic tool determines based on the firm's current location on the profit function.

Insert Figure 2 and Table 8 about here

Applying this diagnostic tool to companies $k = 1, \dots, K$ (in each of the two segments across the four years), we generate the following empirical results. Panel A of Table 8 displays the average proportions of firms per year in each of the two segments which are located on the uphill side of the profit function (U-type for under-investing), downhill side (D-type for over-investing), or near optimal (N-type) with respect to their current levels of investments in news quality, distribution, and advertising sales efforts. Panel B of Table 8 presents the proportions of U-, D- or N-type firms with respect to each investment variable in the 2 segments for the years 1998-2001. (It is important to note that when firms invest in *multiple* marketing activities such

as in our study, the typology of uphill, downhill and near-optimal refers to a particular investment decision by that company. That is, in practice, the same company can be under-investing in quality, over-investing in distribution, near the optimum spending on advertising (type-N), or some other combination.).

Focusing on Panel A of Table 8, we see that the majority of the newspapers across the four years — over 60% in the case of the Small Firm segment and over 80% in the Large Firm segment — are of the N-type with respect to all three (q, d, a) investments. The higher proportion of N-type firms in the Large Segment suggests they may possess greater expertise in gauging marketing-sales response relationships. Next, when firms are suboptimal with respect to quality investments they tend to be underspending rather than overspending. This tendency is more pronounced in the case of Small Firm newspapers. Similarly, we see that in both segments, the majority of suboptimal firms are underspending with respect to their advertising sales investments, specifically, 33.3 % of the Small Firm segment and nearly 14.7% of the Large Firm segment. In contrast, in the case of distribution investments, we find a greater proportion of suboptimal firms in the Small Firm segment overspends rather than underspends. This finding reinforces our earlier inference that Small Firm's circulation-distribution investments are closer to the market saturation levels.

Panel B of Table 8 indicates the above patterns of results hold across the years except 1998, where a significant proportion of suboptimal firms over-invested in ad space sales efforts. This outcome may arise because some firms mistakenly assumed that they were operating in market type 4 when in fact the market was of type 2 for the Small Firm segment. Also we see that significantly greater proportions of Large Firm segment firms were sub-optimal (underinvesting) in all their marketing investments in 1999, which may be attributable to fears of

an economic slowdown and recession-like conditions in 1999. In contrast, greater proportions of firms in both segments appear to be overinvesting in news quality in 2001.

Summarizing the findings across the Panels A and B, we observe that the two segments differ in their spending tendencies as follows:

Investment	<i>Small Firm</i>	<i>Large Firm</i>
Under-spenders in Quality	More likely	Less Likely
Under-spenders in Distribution	Less likely	More Likely
Under-spenders in Advertising	More likely	Less Likely

These findings—a majority of the newspapers in both the segments are N-type with respect to all three investment variables, and greater proportions of suboptimal firms with respect to quality and advertising sales investments are of the U-type than D-type—run counter to previous research results that characterize marketing managers as overspenders (e.g., Aaker and Carman 1982; Prasad and Sen 1999; Hanssens et al. 2001, p. 363). They, however, lend support to and comport with the principles of economic theory that expect firms to behave optimally, especially in a hundred-year old newspaper industry where many suboptimal firms would have disappeared. Moreover, the specific finding that the majority of newspapers are either nearly-optimal or underspending with respect to quality substantiates Meyer and Kim’s (2005, p, 7) conjecture that these companies—especially the smaller ones—are “clustered on the uphill” of the profit function with respect to news quality investments. Finally, the changing proportions of N-type firms across the years reveal that managers would find it difficult to know their company’s location on the profit function in the absence of the proposed diagnostic tool. Consequently, they risk making serious errors in investment decisions described at the outset of this paper that essentially, if unwittingly, lead to liquidating their businesses as feared by

industry analysts (e.g., Rosenstiel and Mitchell 2004). The application of our proposed algorithm to locate their positions on the profit function would assist them in making appropriate investment decisions (i.e., increase, decrease, or maintain status quo) and thus steer their companies to regions of enhanced profitability.

Discussion and Conclusions

Based on our analysis of the Inland Press Association data, the four key takeaways from this paper are as follows: (i) daily newspapers' dual revenues are interrelated with positive feedback effects; this is a common assumption in theoretical models but backed by limited empirical evidence to date. (ii) Investments in news quality not only impact subscription sales directly, but also advertising revenues via subscriptions indirectly. This result is especially true for smaller circulation newspapers whose newsrooms and editorial departments tend to be understaffed and overworked. Consequently, our answer to the question "Is good news quality good business?" is a resounding "Yes." (iii) Advertising revenues of both small and large firms positively respond to efforts to directly boost circulation, e.g., investment in news quality, as much as they respond to advertising space selling effort. (iv) Investments in circulation-distribution also have significant direct and indirect effects on subscriptions and advertising revenues respectively, although the strengths of these effects are not as high as those of news quality investments.

Collectively, these findings reinforce the value of marketing by establishing that marketing investments influence a firm's marketplace performance. More importantly, managers should recognize the assets built by these investments (see Rust, Ambler, Carpenter, Kumar, and Srivastava 2004). As already mentioned, the Newspaper Association of America,

has noted that delivery systems built by investments in circulation-distribution are significant strategic assets (see NAA Task Force Report 2000). Similarly, investments in news quality and advertising sales enhance perceived quality and awareness, thereby building brand equity, an invaluable marketing asset (see Keller 1998).

More generally, this paper has examined and derived normative marketing-mix decision rules in dual revenues markets with interrelated demands, generalizing the classical theorem due to Dorfman and Steiner (1954) and enriching the understanding of resource allocation across four different market types. Further, the empirical analysis of the impact of marketing investments yielded elasticity estimates, augmenting the sparse literature on quality, distribution and personal selling effort elasticities. Although we ignored marketing carryover effects, we note that the directionality of theoretical results would remain unaltered. Moreover, when carryover effects are considered, the optimal profit-maximizing investment levels are *larger* than the short-term optima. In other words, our current findings—that newspaper firms tend to underspend with respect to quality and advertising effort— would only be *reinforced*. Thus, the major conclusions from our research are conservative.

Finally, as the title states, a key contribution of this article is the diagnostic tool, based on a simple 5-step algorithm, which extracts information contained in the market data— on the responsiveness of readers and advertisers, and their interrelated demands — and combines this information with the firm’s knowledge of margins to not only recommend appropriate investments in quality, distribution and advertising, but also identify an individual firm’s location on the profit function to mitigate under- or overspending errors. In addition, it offers guidance by tracing the optimal path that drives the company from its current location to the neighborhood of the maximum profit (see Panel C, Figure 2). Recently, the chief executive of the world’s

largest ad agency WPP Group, Sir Martin Sorrell, stated in a *Wall Street Journal* interview that “... scientific analysis, including econometrics, is one of the most important areas in the marketing-services industry” (Patrick 2005). The diagnostic tool we have developed belongs to this genre of sophisticated econometric approaches. We hope managers find it useful to enhance their firm’s profitability.

Appendix: Derivation of the optimal investments in quality, distribution and advertising

Note that equations (1) through (3) specify the two demand functions and the resulting profit. Let (q^*, d^*, a^*) denote the three optimal investment levels, which exist when the profit function is twice differentiable and its Hessian matrix is negative definite over the set \mathfrak{T} , where $\mathfrak{T} = \{(q, d, a) \in \mathbb{R}^3 \mid q > 0, d > 0, a > 0\}$.

Further, let π_x denote the partial derivative of profit with respect to variable x ($x = q, d, a$). The first-order conditions (FOCs) that maximize profit in (3) are as follows:

$$(A1) \quad \pi_q = km_1 \frac{\partial S}{\partial q} + m_2 \frac{\partial R}{\partial q} - 1 = 0,$$

$$(A2) \quad \pi_d = km_1 \frac{\partial S}{\partial d} + m_2 \frac{\partial R}{\partial d} - 1 = 0, \text{ and}$$

$$(A3) \quad \pi_a = km_1 \frac{\partial S}{\partial a} + m_2 \frac{\partial R}{\partial a} - 1 = 0.$$

To derive the optimal investment levels, consider first the change in subscription sales due to a marginal increase in quality. Formally,

$$(A4) \quad \frac{\partial S}{\partial q} = \frac{\partial f_1}{\partial q} + \frac{\partial f_1}{\partial R} \frac{\partial f_2}{\partial S} \frac{\partial S}{\partial q} \Rightarrow \frac{\partial S}{\partial q} = \frac{1}{\delta} \frac{\partial f_1}{\partial q}, \text{ where } \delta = 1 - \frac{\partial f_1}{\partial R} \frac{\partial f_2}{\partial S}.$$

Similarly, marginal increases in distribution and advertising investments yield the increased subscriptions:

$$(A5) \quad \frac{\partial S}{\partial d} = \frac{1}{\delta} \frac{\partial f_1}{\partial d}, \text{ and}$$

$$(A6) \quad \frac{\partial S}{\partial a} = \frac{1}{\delta} \frac{\partial f_1}{\partial a} \frac{\partial R}{\partial a}.$$

Next, consider the impact on advertising revenues due to a marginal increase in quality, which is given by

$$(A7) \quad \frac{\partial R}{\partial q} = \frac{\partial f_2}{\partial S} \frac{\partial f_1}{\partial q} + \frac{\partial f_2}{\partial S} \frac{\partial f_1}{\partial R} \frac{\partial R}{\partial q} \Rightarrow \frac{\partial R}{\partial q} = \frac{1}{\delta} \frac{\partial f_2}{\partial S} \frac{\partial f_1}{\partial q}.$$

Similarly, marginal increases in distribution and advertising yield the increased ad revenues:

$$(A8) \quad \frac{\partial R}{\partial d} = \frac{1}{\delta} \frac{\partial f_2}{\partial S} \frac{\partial f_1}{\partial d}, \text{ and}$$

$$(A9) \quad \frac{\partial R}{\partial a} = \frac{1}{\delta} \frac{\partial f_2}{\partial a}.$$

Finally, by substituting (A4) through (A9) in (A1), (A2) and (A3), we obtain the simplified first-order conditions:

$$\begin{aligned}
 & m_1 k \frac{\partial f_1}{\partial q} + m_2 \frac{\partial f_2}{\partial S} \frac{\partial f_1}{\partial q} = \delta \\
 (A10) \quad & m_1 k \frac{\partial f_1}{\partial d} + m_2 \frac{\partial f_2}{\partial S} \frac{\partial f_1}{\partial d} = \delta \\
 & m_1 k \frac{\partial f_1}{\partial R} \frac{\partial f_2}{\partial a} + m_2 \frac{\partial f_2}{\partial a} = \delta.
 \end{aligned}$$

The presence of $\delta \neq 1$ in (A10) makes these FOCs different from the standard ones obtained in single-revenue markets (e.g., Dorfman and Steiner 1954).

FIGURE 1.
Firm Types Based on the Locations on a Profit Function

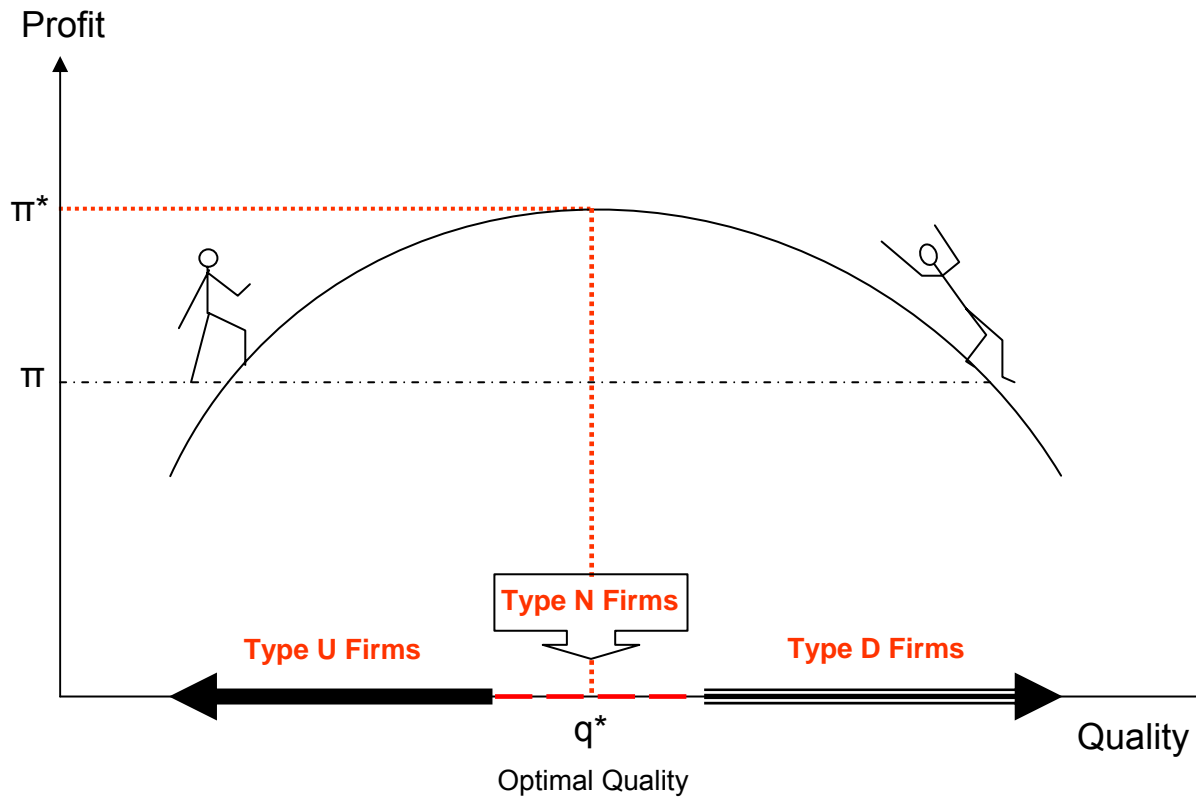
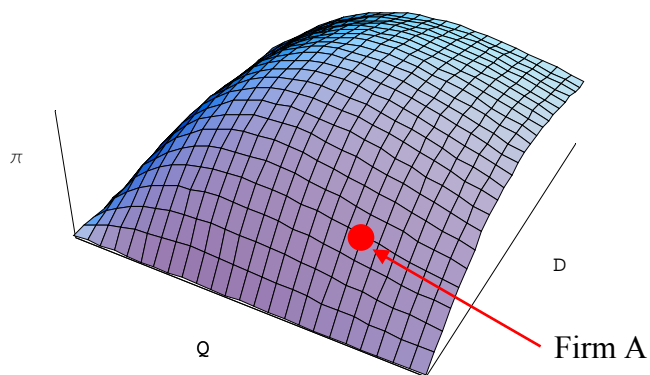
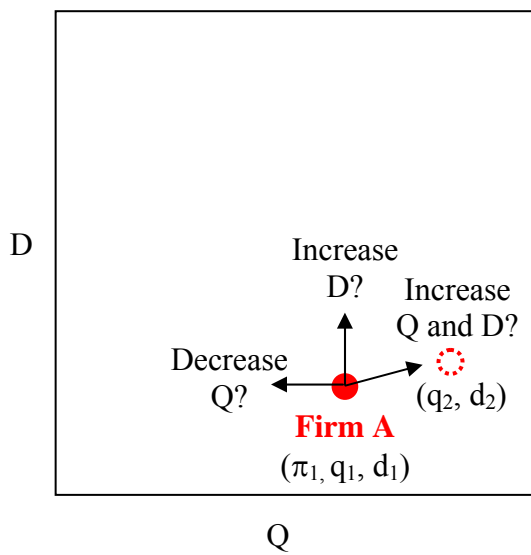


FIGURE 2.
How the Diagnostic Tool Works

Panel A: Firm on a Profit Function



Panel B: Business Reality



Panel C: Diagnostic Tool

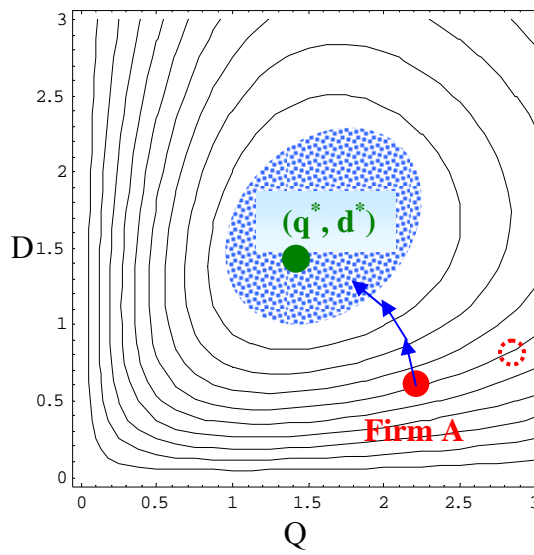


TABLE 1.
Investment Decision Rules Across the Four Market Types

<i>Market Types, j</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
Characteristics	Unrelated Markets	Partially Related Markets	Interrelated Markets with Opposing Feedback	Interrelated Markets with Positive Feedback
Conditions	$\partial f_1/\partial R = 0;$ $\partial f_2/\partial S = 0$	$\partial f_1/\partial R = 0;$ $\partial f_2/\partial S > 0$	$\partial f_1/\partial R < 0;$ $\partial f_2/\partial S > 0$	$\partial f_1/\partial R > 0;$ $\partial f_2/\partial S > 0$
Cross-market dependency	$\delta = 1$	$\delta = 1$	$\delta > 1$	$\delta < 1$
Quality Decisions	q_1^*	$q_2^* > q_1^*$	$q_3^* < q_2^*;$ $q_3^* > q_1^*$ <i>iff</i> $m_2/(km_1) > \partial f_1/\partial R $	$q_4^* > q_2^* > q_1^*;$
Distribution Decisions	d_1^*	$d_2^* > d_1^*$	$d_3^* < d_2^*;$ $d_3^* > d_1^*$ <i>iff</i> $m_2/(km_1) > \partial f_1/\partial R $	$d_4^* > d_2^* > d_1^*;$
Advertising Decisions	a_1^*	$a_2^* = a_1^*$	$a_3^* < a_2^*$	$a_4^* > a_2^* = a_1^*$

TABLE 2.
Model Selection Based on Mixture Regression Criterion

Year	1 Segment	2 Segments	3 Segments
1998	3263.6	3203.83	3255.21
1999	4491.3	4483.3	5554.2
2000	3768.5	3751.68	3806.6
2001	3671.87	3644.8	3649.1

Table 3.
Descriptive Statistics of Cluster Indicators

	<i>Segment 1 (Small Firm)</i>		<i>Segment 2 (Large Firm)</i>	
	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>
Year 1998				
Segment Size (%)	57		43	
TOTSAL (\$)	998052	64775	3992457	224826
NEMP	19	1.04	53	2.2
AEMP	13	0.65	38	1.8
DEMP	9	0.63	39	2.5
Year 1999				
Segment Size (%)	52		48	
TOTSAL	945774	41540	4069345	163764
NEMP	17	0.66	53	1.69
AEMP	13	0.49	41	1.91
DEMP	8	0.43	37	1.71
Year 2000				
Segment Size (%)	55		45	
TOTSAL	1127520	57097	4571215	212355
NEMP	19	0.81	57	2.10
AEMP	14	0.66	40	1.38
DEMP	10	0.54	39	2.10
Year 2001				
Segment Size (%)	69		31	
TOTSAL	1403702	73491	5567286	318934
NEMP	22	0.9	64	3.06
AEMP	17	0.7	45	1.79
DEMP	12	0.65	46	2.75

TABLE 4.
Descriptive Statistics by Segments

<i>SEGMENT 1 (Small Firm)</i>								
<i>Variables</i>	1998(K=140)		1999(K=170)		2000(K=158)		2001(K=193)	
	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>
Quality Investments ('000), q	1058.7	615.1	999.6	548.5	1159.4	612.2	1467.6	842.6
Distribution Investments ('000), d	653.1	416.1	640.0	395.6	780.3	461.1	1022.7	691.4
Ad sales Investments ('000), a	844.6	451.9	807.1	431.3	980.7	552.5	1231.3	740.4
Subscriptions Sales ('000), S	13.9	7.5	12.7	6.4	13.7	6.5	16.3	8.5
Advertising Revenue ('00,000 \$), R	36.1	25.6	33.5	22.1	39.5	25.02	49.9	33.8
Subscription Margin, m ₁	0.17	0.06	0.17	0.06	0.16	0.05	0.17	0.06
Ad Revenue Margin, m ₂	0.71	0.14	0.75	0.14	0.76	0.15	0.77	0.17

<i>SEGMENT 2 (Large Firm)</i>								
<i>Variables</i>	1998(K=107)		1999(K=157)		2000(K=128)		2001(K=85)	
	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>	<i>Mean</i>	<i>Standard Deviation</i>
Quality Investments ('000), q	4054.5	2065.5	4094.9	201.9	4637.7	2222.7	5519.1	2483.5
Distribution Investments ('000), d	3038.9	1819.3	3089.5	173.4	3428.8	1952.7	4284.1	2110.1
Ad sales Investments ('000), a	3025.5	1444.4	3236.6	158.3	3487.8	1511.1	4010.2	1712.1
Subscriptions Sales ('000), S	39.7	16.7	39.35	16.4	41.2	16.2	47.07	16.7
Advertising Revenue ('00,000 \$), R	144.6	80.2	155.8	88.5	169.01	84.3	190.7	85.7
Subscription Margin, m ₁	0.21	0.09	0.21	0.08	0.19	0.08	0.19	0.09
Ad Revenue Margin, m ₂	0.75	0.11	0.79	0.09	0.78	0.11	0.79	0.12

TABLE 5.
Parameter Estimates (t-values) for the Dual Revenues Model

<i>Parameters</i>	<i>1998</i>		<i>1999</i>		<i>2000</i>		<i>2001</i>	
	<i>Segment 1 (Small Firm)</i>	<i>Segment 2 (Large Firm)</i>	<i>Segment 1 (Small Firm)</i>	<i>Segment 2 (Large Firm)</i>	<i>Segment 1 (Small Firm)</i>	<i>Segment 2 (Large Firm)</i>	<i>Segment 1 (Small Firm)</i>	<i>Segment 2 (Large Firm)</i>
Intercept, α_0	-86.9 (-2.49)	-142.8 (-1.60)	-57.51 (-2.32)	-273.1 (-3.33)	-61.06 (-4.06)	-176.28 (-1.65)	-83.58 (-3.67)	-183.4 (-1.74)
Ad Revenue Coefficient, α_1	0.07 (0.94)	0.13 (2.96)	0.12 (1.93)	0.06 (1.69)	0.11 (3.15)	0.09 (2.06)	0.09 (2.32)	0.11 (2.81)
Quality Coefficient, α_2	5.79 (2.54)	5.66 (1.57)	4.13 (2.22)	12.1 (3.06)	3.8 (3.46)	8.5 (1.61)	3.96 (2.63)	8.94 (1.57)
Distribution Coefficient, α_3	1.41 (1.38)	5.25 (1.59)	0.73 (1.39)	8.02 (2.92)	1.32 (2.31)	4.77 (1.69)	2.92 (3.60)	4.63 (1.91)
Intercept, β_0	-147.3 (-3.49)	-467.2 (-1.75)	-103.1 (-2.92)	-554.3 (-3.60)	-165.04 (-4.52)	-473.33 (-2.25)	-232.59 (-4.68)	-660.11 (-2.1)
Subscription Coefficient, β_1	2.38 (7.47)	3.83 (6.23)	2.67 (8.30)	4.16 (10.60)	2.4 (6.91)	4.55 (9.55)	2.49 (7.05)	4.08 (6.61)
Ad sales effort Coefficient, β_2	11.15 (3.25)	30.99 (1.59)	7.6 (2.63)	36.7 (3.28)	12.5 (4.20)	30.34 (1.99)	17.4 (4.39)	43.53 (1.94)
Subscription Model Fit, R^2	0.79	0.80	0.80	0.81	0.83	0.81	0.81	0.82
Ad Revenue Model Fit, R^2	0.86	0.77	0.81	0.79	0.81	0.84	0.82	0.83

TABLE 6.
Determination of Market Type

<i>Parameters</i>	<i>1998</i>		<i>1999</i>		<i>2000</i>		<i>2001</i>	
	<i>SI*</i>	<i>S2*</i>	<i>SI</i>	<i>S2</i>	<i>SI</i>	<i>S2</i>	<i>SI</i>	<i>S2</i>
α_1	0.07	0.13	0.12	0.06	0.11	0.09	0.09	0.11
β_1	2.38	3.83	2.67	4.16	2.4	4.55	2.49	4.08
$\delta = 1 - \alpha_1 \beta_1$	0.83	0.50	0.68	0.75	0.73	0.59	0.77	0.55
Var[δ]	0.043	0.056	0.036	0.19	0.009	0.05	0.01	0.05
t-value	-0.82	-2.11	-1.67	-1.72	-2.81	-1.80	-2.30	-2.04
Market Type	2	4	4	4	4	4	4	4

**SI* and *S2* denote the Small Firm and Large Firm segments, respectively.

TABLE 7.
Investment Elasticities

<i>Investment Elasticity</i>	<i>Expression</i>	<i>1998</i>		<i>1999</i>		<i>2000</i>		<i>2001</i>		<i>S1</i>	<i>S2</i>	<i>Overall Average</i>
		<i>SI**</i>	<i>S2**</i>	<i>SI</i>	<i>S2</i>	<i>SI</i>	<i>S2</i>	<i>SI</i>	<i>S2</i>	<i>Avg.</i>	<i>Avg.</i>	
Subscription wrt* advertising sales, $\partial \text{Ln}(S)/\partial \text{Ln}(a)$	$\frac{\alpha_1 \beta_2}{S(1 - \alpha_1 \beta_1)}$	0.10	0.23	0.14	0.09	0.18	0.13	0.19	0.21	0.15	0.17	0.16
Subscriptions wrt distribution, $\partial \text{Ln}(S)/\partial \text{Ln}(d)$	$\frac{\alpha_3}{S(1 - \alpha_1 \beta_1)}$	0.16	0.30	0.11	0.32	0.17	0.23	0.35	0.20	0.20	0.26	0.23
Subscriptions wrt quality, $\partial \text{Ln}(S)/\partial \text{Ln}(q)$	$\frac{\alpha_2}{S(1 - \alpha_1 \beta_1)}$	0.68	0.33	0.63	0.48	0.50	0.40	0.47	0.39	0.57	0.40	0.49
Ad Revenue wrt advertising sales, $\partial \text{Ln}(R)/\partial \text{Ln}(a)$	$\frac{\beta_2}{R(1 - \alpha_1 \beta_1)}$	0.61	0.54	0.52	0.41	0.64	0.38	0.73	0.50	0.63	0.46	0.54
Ad Revenue wrt distribution, $\partial \text{Ln}(R)/\partial \text{Ln}(d)$	$\frac{\alpha_3 \beta_1}{R(1 - \alpha_1 \beta_1)}$	0.19	0.35	0.13	0.37	0.19	0.28	0.36	0.22	0.22	0.31	0.26
Ad Revenue wrt quality, $\partial \text{Ln}(R)/\partial \text{Ln}(q)$	$\frac{\alpha_2 \beta_1}{R(1 - \alpha_1 \beta_1)}$	0.76	0.38	0.76	0.56	0.53	0.49	0.49	0.42	0.64	0.46	0.55

* wrt abbreviates "with respect to"

** *SI* and *S2* denote the Small Firm and Large Firm segments, respectively.

TABLE 8.
Proportions of Firms Under-investing, Nearly Optimal, or Over-investing

Panel A: Average across years

<i>Segment 1 (Small Firm)</i>			
	Type-U Under-Investing	Type-N Nearly Optimal	Type-D Over-Investing
Quality	22.1	68.7	9.2
Distribution	12.5	68.5	19.0
Advertising	33.3	62.8	3.9

<i>Segment 2 (Large Firm)</i>			
	Type-U Under-Investing	Type-N Nearly Optimal	Type-D Over-Investing
Quality	14.8	82.3	2.9
Distribution	11.5	85.4	3.2
Advertising	14.7	84.0	1.3

Panel B: Year by Year

Investments	<i>Segment 1 (Small Firm)</i>			<i>Segment 2 (Large Firm)</i>		
	Type- U	Type- N	Type-D	Type- U	Type- N	Type-D
Year 1998						
Quality	28.5	71.4	0.1	0.93	98.14	0.93
Distribution	0.7	80.7	18.6	2.8	97.1	0.1
Ad Sales	1.5	80	18.5	6.5	93.4	0.1
Year 1999						
Quality	30.6	65.9	3.5	55.5	42.6	1.9
Distribution	0.6	74.1	25.3	44.5	54.7	0.63
Ad Sales	17.1	77.1	5.8	26.1	71.3	2.5
Year 2000						
Quality	8.2	84.4	6.3	1.5	97.6	0.9
Distribution	0.2	64.5	35.3	0.7	92.9	6.7
Ad Sales	29.7	67.08	3.16	6.2	92.9	0.8
Year 2001						
Quality	6.7	71.3	21.8	1.2	95.2	3.6
Distribution	23.4	58.8	17.7	1.2	94.1	4.7
Ad Sales	51.5	45.3	3.2	24.7	74.1	1.2

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